Synthesis of multi indicator system: *Min - BoD interval*

Starting from a matrix $\mathbf{X_{NxJ}}$, normalized with the Min-Max method, the objective is to obtain an interval $\mathbf{I}$ defined by the **minimum** between the basic indicators of the considered unit and the achievable maximum, obtained with the **Benefit of the Doubt approach**:

$$\mathbf{X} \equiv \begin{Bmatrix} x_{11} & x_{12} & \cdots & x_{1J} \\ x_{21} & x_{22} & \cdots & x_{2J} \\ \vdots & \ddots & \ddots & \vdots \\ x_{N1} & x_{N2} & \cdots & x_{NJ} \end{Bmatrix} \Rightarrow \mathbf{R} \Rightarrow \mathbf{I} \equiv \left\{ I_i : i = 1...N \right\} \equiv \begin{pmatrix} I_1 = [\underline{Min_1}, \overline{BoD_1}] \\ I_2 = [\underline{Min_2}, \overline{BoD_2}] \\ \vdots \\ I_i = [\underline{Min_i}, \overline{BoD_i}] \\ \vdots \\ I_N = [\underline{Min_N}, \overline{BoD_N}] \end{pmatrix}$$

The interval defines a *range of variation of the performance* for each unit, in which each point is hypothetically reachable and the values of *all the mean-based aggregation methods* are included

Synthetic measure selection via GMM

Finite mixture of Gaussians model (GMM) based clustering gives an *unsupervised* and *flexible* classification, i.e. able to approximate the density function of any unknown distribution, that allows us to classify the statistical units into g clusters, $C_1, C_2, \ldots, C_g$, such that the units within each cluster are homogeneous and similar to each other.
Starting from the matrix $\mathbf{X} \equiv \{x_{ij}\}$, let us suppose to construct k synthetic indicators, $_1\mathbf{v}, _2\mathbf{v}, \ldots _k\mathbf{v}$ using k different aggregative methods.
We divide each vector $\mathbf{v}$ into g groups, made up according to the results of the classification and for each cluster, calculate the coefficients of variation, cv, of the k groups of synthetic values:

- *The closer the values obtained for the units classified in a group are one another, the more the method with which they were obtained corresponds to a "good choice"*, since it respects the internal homogeneity of the group and the clustering discriminatory capacity.

Example on Human Development Index data

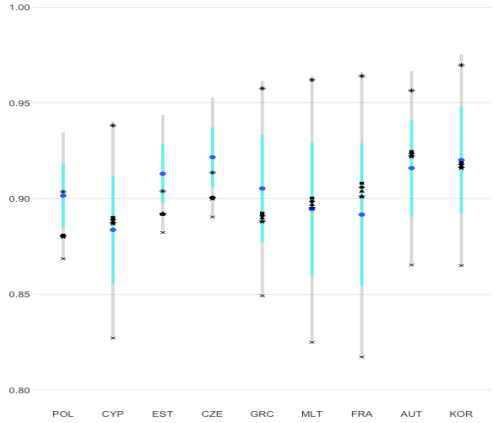

Table: cv of different synthetic measures

	Cluster 1	Cluster 2	Cluster 3	Overall
GM	13.48	8.86	2.50	9.33
AM	12.47	8.60	2.48	8.87
QM	11.64	8.35	2.46	8.47
CM	11.04	8.12	2.44	8.16
Min	21.66	12.26	4.13	13.95
Max	10.31	6.67	2.55	7.17
BoD	10.22	6.56	2.02	7.01