A stochastic blockmodel for interaction lengths

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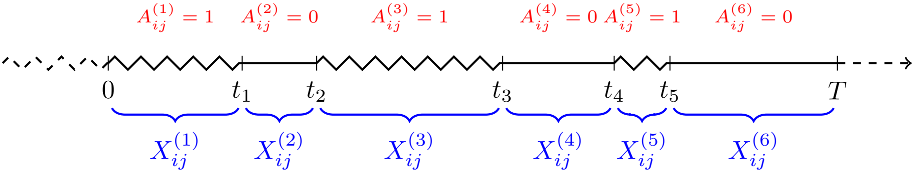
Rastelli, R., & Fop, M.: A stochastic block model for interaction lengths. *Advances for Data Analysis and Classification* (2020).

Data

- We study the **lengths of pairwise interactions** between individuals.
- The *observed data* are a *continuous collection* of **adjacency matrices** $\boldsymbol{\mathcal{E}}(t)$, $\forall t \in [0, T]$, where:

$$\mathcal{E}_{ij}(t) = \begin{cases} 1 & \text{if } i \text{ and } j \text{ are } \textbf{interacting} \text{ at time } t \\ 0 & \text{if } i \text{ and } j \text{ are } \textbf{not interacting} \text{ at time } t \end{cases}$$

Example of **interaction timeline** between individuals i and j:



Models

From a **homogeneous model**...

$$X_{ij}^{(w)} \overset{IID}{\sim} Exp\left(A_{ij}^{(w)}\mu + \left(1 - A_{ij}^{(w)}\right)\nu\right) \qquad \mu \in \mathbb{R}^+,\ \nu \in \mathbb{R}^+$$

($1/\mu$ is the ***average interaction length*** & $1/\nu$ is the ***average non-interaction length***)

$$\ell(\mu,\nu) = \log\mu \sum_{i,j,w} A_{ij}^{(w)} + \log\nu \sum_{i,j,w}\left(1 - A_{ij}^{(w)}\right) - \mu\sum_{i,j,w} A_{ij}^{(w)}X_{ij}^{(w)} - \nu\sum_{i,j,w}\left(1 - A_{ij}^{(w)}\right)X_{ij}^{(w)}$$

... to a **stochastic blockmodel**:

$$X_{ij}^{(w)}\Big| Z_i = g,\ Z_j = h \sim Exp\left(A_{ij}^{(w)}\mu_{gh} + \left(1 - A_{ij}^{(w)}\right)\nu_{gh}\right)$$

$$\ell(\boldsymbol{\mu},\boldsymbol{\nu},\mathbf{Z}) = \sum_{g=1}^{K}\sum_{h=1}^{K}\left\{\mathcal{A}_{gh}^{(+)}\log(\mu_{gh}) + \mathcal{A}_{gh}^{(-)}\log(\nu_{gh}) - \mathcal{X}_{gh}^{(+)}\mu_{gh} - \mathcal{X}_{gh}^{(-)}\nu_{gh}\right\}$$

$$\mathcal{A}_{gh}^{(+)} = \sum_{i,j,w} Z_{ig}Z_{jh}A_{ij}^{(w)} \qquad \mathcal{A}_{gh}^{(-)} = \sum_{i,j,w} Z_{ig}Z_{jh}\left(1 - A_{ij}^{(w)}\right)$$

$$\mathcal{X}_{gh}^{(+)} = \sum_{i,j,w} Z_{ig}Z_{jh}A_{ij}^{(w)}X_{ij}^{(w)} \qquad \mathcal{X}_{gh}^{(-)} = \sum_{i,j,w} Z_{ig}Z_{jh}\left(1 - A_{ij}^{(w)}\right)X_{ij}^{(w)}$$

Inference & Application

Inference: Variational Expectation-Maximisation algorithm + Integrated Completed Likelihood.

Package `expSBM` available from **CRAN**.

London bikes dataset: Two stations are interacting if there is at least one bike moving from one to the other.