On MORAN's Property of the Poisson Distribution

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Abstract

Two interesting results encountered in the literature concerning the Poisson and the negative binomial distributions are due to MORAN (1952) and PATIL & SESHADRI (1964), respectively.

MORAN's result provided a fundamental property of the Poisson distribution. Roughly speaking, he has shown that if Y, Z are independent, non-negative, integer-valued random variables with $X = Y | Z$ then, under some mild restrictions, the conditional distribution of $Y | X$ is binomial if and only if Y, Z are Poisson random variables.

Motivated by MORAN's result PATIL & SESHADRI obtained a general characterization. A special case of this characterization suggests that, with conditions similar to those imposed by MORAN, $Y | X$ is negative hypergeometric if and only if Y, Z are negative binomials.

In this paper we examine the results of MORAN and PATIL & SESHADRI in the case where the conditional distribution of $Y | X$ is truncated at an arbitrary point $k-1$ ($k = 1, 2, \dots$). In fact we attempt to answer the question as to whether MORAN's property of the Poisson distribution, and subsequently PATIL & SESHADRI's property of the negative binomial distribution, can be extended, in one form or another, to the case where $Y | X$ is binomial truncated at $k-1$ and negative hypergeometric truncated at $k-1$ respectively.

Key words and Phrases: Poisson Distribution, Binomial Distribution, Negative Binomial Distribution, Negative Hypergeometric Distribution, MORAN's Theorem, PATIL & SESHADRI's Theorem.