CHARACTERIZATION OF DISCRETE DISTRIBUTIONS BASED ON CONDITIONALITY AND DAMAGE MODELS

Contribution to the Theory of Discrete
Univariate and Multivariate Distributions,
with Particular Emphasis on Conditionality
Characterizations Using Methods Related
to the Rao-Rubin Property.

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A Thesis Submitted to the Postgraduate School of Mathematics,
University of Bradford for the Degree of
DOCTOR OF PHILOSOPHY.

1977.

To Kiki

PREFACE

Let X and Y be two non-negative, integer-valued random variables, such that $X \geqslant Y$, and let $Z=X-Y$. When the conditional distribution of $Y|X=n$ is used for making inferences about the distribution of X or the distribution of Y, this model is called a conditionality model.

Rao (Classical and Contagious discrete distributions 1963), introduced a new version of the conditionality model; he called this a damage model. In this model X represents an observation which is produced by some natural process and which may be partially damaged; $Y|X=n$ is the destructive process. Thus Y stands for what we actually observe of X (the remaining part of X).

Rao and Rubin (Sankhyā 1964) obtained a characterization for the Poisson distribution using damage model theory and a condition which has come to be known as the Rao-Rubin condition.

In this thesis an extension of the Rao-Rubin characterization which has been suggested by the work of Shanbhag (1976) has been used to characterize many well-known discrete distributions as the distribution of X or as the distribution of $Y|X=n$ when the other one of the two is given. This model is extended to provide characterizations for truncated distributions.

A new model is suggested enabling us to characterize finite discrete distributions, truncated and untruncated.

Bivariate and Multivariate extensions of all the results obtained in the Univariate case are derived.

Finally, the damage model is examined in the more general situation where either the distribution of X or the distribution of $Y|X=n$ is a compound distribution. Some interesting characterizations are provided by this situation.

Many of the results existing in the literature in this field are found to be special cases of our results.

ACKNOWLEDGEMENTS

I would like to express my gratitude to Dr A.W. Kemp for all her efforts, advice and assistance during the preparation of this thesis. I am also grateful to Professor C.D. Kemp for his continued encouragement and willingness to contribute to the accomplishment of my goal.

I would like to thank Dr D.N. Shanbhag for the motivation he has provided me through numerous constructive and stimulating discussions on the subject.

I am extemely thankful to Miss Kiki Xekalaki for her invaluable comments and suggestions concerning many aspects of my research.

I am deeply indebted to my parents and my sister for their constant support and encouragement throughout my studies. My sister, Miss F. Panaretos, deserves special thanks for checking the final corrections of the typescript.

Finally, I am particularly appreciative of the intelligent effort put in by Mrs Lesley Barratt in typing the manuscript patiently and skillfully.

TABLE OF CONTENTS