CHAPTER 5.

CHARACTERIZATIONS OF FINITE DISCRETE DISTRIBUTIONS.

5.0 Introduction

In Chapters 3 and 4 we used theorem 3.1.1 and its variants and extensions to obtain characterizations for a number of Univariate and Multivariate Distributions, based on the R-R condition. In this Chapter we introduce another possible form of relation between the distributions of Y and Y|X. Making use of this relationship we obtain a general characterization for a class of "modified" distributions with finite range. This class includes distributions, the first m probabilities of which, are given by a certain law and the rest N-m (N being the range of the values of the r.v. X) are arbitrary constants. (The Binomial distribution is a member of that family if one considers m=N). An extension to this result is also given which makes possible characterizations of truncated forms of the distribution of X. Finally characterizations based on truncated forms of the conditional distribution of Y|X are obtained.

5.1 The Univariate Case

Theorem 5.1.1

Let $\{(a_n, b_n)\ n=0,1,\dots\}$ be a sequence of real vectors such that

$$\begin{aligned} &a_n > 0,\ n=0,1,\dots,m, \text{ and } a_n = 0,\ n=m+1,\dots,N \\ &b_n > 0,\ n=0,1,\dots,\ell, \\ &\ell \text{ fixed},\ 1 \leqslant \ell \leqslant N-m,\quad 0 < m \leqslant N,\quad N > 0. \end{aligned} \tag{5.1.1}$$

and let $\left\{\sum_{r=0}^{n} a_r b_{n-r}\ n=0,1,\dots,N\right\}$ be denoted by $\{c_n : n=0,1,\dots,N\}$.

Let $\{X,Y\}$ be a random vector of non-negative integer-valued components such that X has the distribution

$$P\{X=n\} = P_n \quad n=0,1,\dots,N \text{ with } P_n > 0 \text{ for } 0 \leqslant n \leqslant \ell,\ \ell \geqslant 1.$$

Suppose that whenever $P_n > 0$

$$P(Y=r|X=n) = \frac{a_r b_{n-r}}{c_n} \qquad r=0,1,\dots,n. \tag{5.1.2}$$

Then,

$$P(Y=r|X=Y) = P(Y=r|X=Y+1) = \dots = P(Y=r|X=Y+\ell) \tag{5.1.3}$$

$$r=0,1,\dots,N, \quad (\ell \text{ equations}),$$

for a fixed ℓ $(1 \leqslant \ell \leqslant N-m)$,

iff

$$P_n = \begin{cases} P_0 \dfrac{c_n}{c_0} \theta^n & \text{for some } \theta > 0 \text{ if } n=0,1,\dots,m+\ell \\ d_n & \text{if } m+\ell < n \leqslant N, \end{cases} \tag{5.1.4}$$

where d_n are arbitrary constants with the condition

$$\sum_{n=0}^{\ell+m} P_n + \sum_{n=\ell+m}^{N} d_n = 1.$$

<u>Proof</u> We consider first the "only if" part of the proof. Obviously (5.1.4) is true for n=0. Let us consider the relation

$$P(Y=r|X=Y) = P(Y=r|X=Y+1) \qquad r=0,1,\dots,N\ . \tag{5.1.5}$$

From (5.1.2)

$$P(Y=r|X=Y) = \frac{P_r \dfrac{a_r b_0}{c_r}}{P(X=Y)} \quad \text{and} \tag{5.1.6}$$

$$P(Y=r|X=Y+1) = \frac{P_{r+1} \frac{a_r b_1}{c_{r+1}}}{P(X=Y+1)} . \tag{5.1.7}$$

Also (5.1.5), (5.1.6) and (5.1.7) give

$$P_r \frac{a_r b_0}{c_r} = \frac{P(X=Y)}{P(X=Y+1)} \frac{P_{r+1}}{c_{r+1}} a_r b_1 .$$

Since $a_r \neq 0$ $r=0,1,\ldots,m$

$$\frac{P_{r+1}}{c_{r+1}} = \frac{b_0}{b_1} \frac{P(X=Y+1)}{P(X=Y)} \frac{P_r}{c_r}$$

i.e.

$$\frac{P_{r+1}}{c_{r+1}} = \theta \frac{P_r}{c_r} \quad r=0,1,\ldots,m , \tag{5.1.8}$$

where

$$\theta = \frac{b_0}{b_1} \frac{P(X=Y+1)}{P(X=Y)} .$$

(5.1.8) can be written in the form of the following system of equations.

$$\begin{aligned} \frac{P_1}{c_1} &= \theta \frac{P_0}{c_0} \\ \frac{P_2}{c_2} &= \theta \frac{P_1}{c_1} \\ &\vdots \\ \frac{P_{r+1}}{c_{r+1}} &= \theta \frac{P_r}{c_r} \qquad r \leqslant m. \end{aligned} \tag{5.1.9}$$

Consequently

$$\frac{P_{r+1}}{c_{r+1}} = \theta^{r+1} \frac{P_0}{c_0} \qquad r=0,1,\ldots m, \tag{5.1.10}$$

which means that (5.1.4) is valid for $n=1,\dots,m+1$. We will prove that it is also valid for $n=m+2$. In

$$P(Y=r|X=Y+1) = P(Y=r|X=Y+2) \tag{5.1.11}$$

we have

$$P(Y=r|X=Y+2) = \frac{P_{r+2}\dfrac{a_r b_2}{c_{r+2}}}{P(X=Y+2)} \, . \tag{5.1.12}$$

Also (5.1.7), (5.1.11), (5.1.12) give,

$$\frac{P_{r+2}}{c_{r+2}} = \theta' \frac{P_{r+1}}{c_{r+1}} \qquad r=0,1,\dots,m \tag{5.1.13}$$

with

$$\theta' = \frac{b_1}{b_2}\,\frac{P(Y=Y+2)}{P(X=Y+1)} \, .$$

So, (5.1.13) can be expressed as

$$\begin{aligned} \frac{P_2}{c_2} &= \theta' \frac{P_1}{c_1} \\ \frac{P_3}{c_3} &= \theta' \frac{P_2}{c_2} \\ &\vdots \\ \frac{P_{r+2}}{c_{r+2}} &= \theta' \frac{P_{r+1}}{c_{r+1}} \qquad r=0,1,\dots,m. \end{aligned} \tag{5.1.14}$$

From the second equation in the system (5.1.9) and the first in (5.1.14) we find that $\theta' = \theta$. So, (5.1.14) becomes

$$\frac{P_{r+2}}{c_{r+2}} = \theta^{r+1} \frac{P_1}{c_1} \qquad r=0,1,\dots,m,$$

and taking into consideration (5.1.10) $(r=0)$ we have

$$\frac{P_{m+2}}{c_{m+2}} = \theta^{m+2} \frac{P_0}{c_0} . \tag{5.1.15}$$

Continuing in the same way, from

$$P(Y=r|X=Y+\ell-1) = P(Y=r|X=Y+\ell)$$

(having taken note of all other equalities in (5.1.3)) we derive that

$$\frac{P_{r+\ell}}{c_{r+\ell}} = \theta^{r+\ell} \frac{P_0}{c_0} \qquad r=0,1,\ldots,m. \tag{5.1.16}$$

As a result of (5.1.16), (5.1.4) is established.

Next let us consider the "if" part of the proof. For $0 \leqslant j \leqslant \ell$ we have from (5.1.2) and (5.1.4)

$$P(Y=r|X=Y+j) = \frac{P(Y=r,X=r+j)}{P(X-Y=j)} = \frac{\frac{a_r b_j}{c_{r+j}} P_{r+j}}{P(X-Y=j)}$$

$$= \frac{b_j a_r}{P(X-Y=j)} \frac{P_0}{c_0} \theta^{r+j} \qquad r=0,1,\ldots,m$$

(since $r+j \leqslant m+\ell$).

Hence

$$P(Y=r|X=Y+j) = \frac{a_r \theta^r}{\phi(j,\theta)} \qquad r=0,1,\ldots,m. \tag{5.1.17}$$

Furthermore, the L.H.S. of (5.1.17) is a probability distribution. So,

$$\sum_{r=0}^{m} \frac{a_r \theta^r}{\phi(j,\theta)} = 1$$

i.e.

$$\phi(j,\theta) = \sum_{r=0}^{m} a_r \theta^r = A(\theta). \tag{5.1.18}$$

(5.1.17) and (5.1.18) give

$$P(Y=r|X=Y+j) = \frac{a_r \theta^r}{A(\theta)} \qquad r=0,1,\ldots,m . \qquad (5.1.1$$

Since the R.H.S. of (5.1.19) does not depend on j, we see that $P(Y=r|X-Y=j)$ are independent of j for $j=0,1,\ldots,\ell$ and hence these are all equal for a given r $(r=0,1,\ldots,N)$. So (5.1.3) is established.

5.2 Characterizations of Some Univariate Distributions based on Theorem 5.1.1

As a result of theorem 5.1.1, the following corollaries can be stated, yielding characterizations for a "modified" Binomial and Hypergeometric dist.

Corollary 5.2.1 (Characterization of a "modified" Binomial Distribution)

Consider the random vector (X,Y) as in theorem 5.1.1. Suppose that

$$P(Y=r|X=n) = \frac{\binom{m}{r}\binom{N-m}{n-r}}{\binom{N}{n}} \qquad r \leq n, \quad m,n,N > 0 \quad m \leq N \qquad (5.2.1)$$

i.e. Hypergeometric (N,m,n). Then

$$P(Y=r|X=Y) = P(Y=r|X=Y+1) = \ldots = P(Y=r|X=Y+\ell) \qquad (5.2.2)$$
ℓ fixed, $1 \leq \ell \leq N-m$.

iff

$$P_n = C\binom{N}{n} p^n q^{N-n} \qquad 0 < p < 1 \qquad q = 1-p \qquad (5.2.3)$$

where C is a constant; $N > 0$, $n=0,1,\ldots,\ell+m$.

Proof

Define

$$a_n = \binom{m}{n}, \qquad b_n = \binom{N-m}{n} \qquad n=0,1,\ldots. \qquad (5.2.4)$$

Then,

$$c_n = \sum_{r=0}^{n} a_r b_{n-r} = \binom{N}{n} .$$

These sequences can be used to express (5.2.1) in the form $a_r b_{n-r} / c_n$ and also satisfy the requirements of theorem 5.1.1. Hence, (5.2.2) is equivalent to (5.1.4), which gives, for $n=0,1,\ldots,m+\ell$

$$P_n = P_0 \binom{N}{n} \theta^n \quad \text{i.e.} \quad P_n = C\binom{N}{n} \frac{\theta^n}{(1+\theta)^N} = C\binom{N}{n} p^n q^{N-n} \text{ where } p = \frac{\theta}{1+\theta}$$

and $C = P_0(1+\theta)^N$.

Corollary 5.2.2 (Characterization of the Hypergeometric Distribution)

Suppose that (X,Y) are as in 5.2.1, P_n is Binomial as in (5.2.3), with $\ell=N-m$, $P(Y=r|X=n)$ is of the form (5.1.2). Then condition (5.2.2), for $\ell=N-m$, holds iff $P(Y=r|X=n)$ is Hypergeometric as in (5.2.1).

Proof The "if" part is contained in corollary 5.2.1. For the "only if" part we have from theorem 5.1.1 that (5.2.2), for $\ell=N-m$, holds iff

$$c_n = \frac{P_n}{P_0} c_0 \theta^{-n} = c_0 \frac{\binom{N}{n} p^n q^{N-n}}{q^N} \theta^{-n} = c_0 \binom{N}{n} \left(\frac{p}{q\theta}\right)^n \qquad (5.2.5)$$

Hence

$$\sum_{n=0}^{N} c_n = c_0 \sum_{n=0}^{N} \binom{N}{n} \left(\frac{p}{q\theta}\right)^n = c_0 \left(1 + \frac{p}{q\theta}\right)^N$$

so,

$$c_0 = \sum_{n=0}^{N} c_n \left(1 + \frac{p}{q\theta}\right)^{-N} \qquad (5.2.6)$$

Substituting (5.2.6) in (5.2.5) we get

$$\frac{c_n}{\sum_{n=0}^{N} c_n} = \binom{N}{n}\left(\frac{p}{q\theta}\right)^n \left(1+\frac{p}{q\theta}\right)^{-N} = \binom{N}{n}\pi^n (1-\pi)^{N-n} \tag{5.2.7}$$

where $0 < \pi < 1$ for a suitable $\theta > 0$.

So, the distribution $c_n \Big/ \sum_{n=0}^{N} c_n$ is of the Binomial form. On the other hand it is well known (see e.g. Ramachandran (1961) also in (1967)) that the Binomial distribution is uniquely decomposable into two Binomials with the same probability of success. That implies

$$\frac{a_r}{\sum_{j=0}^{m} a_j} = \binom{m}{r}\pi^r (1-\pi)^{m-r} \qquad r=0,1,\ldots,m \tag{5.2.8}$$

and

$$\frac{b_n}{\sum_{j=0}^{N-m} b_j} = \binom{N-m}{n}\pi^n (1-\pi)^{N-m-n} \qquad n=0,1,\ldots,N-m\ . \tag{5.2.9}$$

The result follows if in $P(Y=r|X=n) = a_r b_{n-r}/c_n$ we let c_n, a_n, b_n be as in (5.2.7), (5.2.8) and (5.2.9) respectively.

Note 1 Patil and Ratnaparkhi (1975) have shown that if the conditional distribution of $Y|X$ is Hypergeometric as in (5.2.1) and the distribution of X is Binomial as in (5.2.3) then the R-R condition $P(Y=r) = P(Y=r|X=Y)$ holds. It can be observed that their result is a side result of the "only if" part of Corollary 5.2.1.

In the same paper Patil and Ratnaparkhi mention that they are investigating the problem of proving that when $Y|X \sim$ Hypergeometric and the R-R condition holds, then X is Binomial.

The counter-example which follows shows that such a result does not hold. Let us consider

$$P(Y=r|X=n) = \frac{\binom{1}{r}\binom{N-1}{n-r}}{\binom{N}{n}} \quad . \quad r=0,1. \tag{5.2.10}$$

i.e. Hypergeometric (N,1,n),

and a probability distribution $P(X=n) = P_n$ such that

$$P_n = \begin{cases} \frac{N^2}{2N^2+N-1} & \text{for } n=0,1 \\ \frac{N-1}{2N^2+N-1} & \text{for } n=N \end{cases} \tag{5.2.11}$$

(Clearly P_n, n=0,1, N is well defined by (5.2.11).)

In the above situation the R-R condition

$$P(Y=r) = P(Y=r|X=Y) \qquad r=0,1 \tag{5.2.12}$$

is equivalent to

$$P(Y=0) = P(Y=0|X=Y) \tag{5.2.13}$$

(5.2.10) and (5.2.13) give

$$\sum_{n=0}^{\infty} P_n \frac{\binom{N-1}{n}}{\binom{N}{n}} = \frac{P_0}{P_0 + \frac{P_1}{N}} \quad . \tag{5.2.14}$$

We may now observe that

$$\frac{P_0}{P_0 + \frac{P_1}{N}} = \frac{N}{N+1} \tag{5.2.15}$$

and

$$\sum_{n=0}^{\infty} P_n \frac{\binom{N-1}{n}}{\binom{N}{n}} = \sum_{n=0}^{N} P_n \frac{N-n}{N}$$

$$= \frac{N^2}{2N^2+N-1}\left(1+\frac{N-1}{N}\right) = \frac{N(2N-1)}{2N^2+N-1} = \frac{N}{N+1} . \qquad (5.2.16)$$

Since the R.H.S. of (5.2.15) and (5.2.16) are identical it is clear that P_n, $n=0,1,N$ as defined in (5.2.11) satisfy the R-R condition.

Hence, there exists a probability distribution other than the Binomial which satisfies the R-R condition.

Note Corollary 5.2.1 implies that in the case mentioned the minimum condition require for P_n to be Binomial is

$$P(Y=r|X=Y) = P(Y=r|X=Y+1) = \ldots = P(Y=r|X=Y+N-m)$$

5.3 The Extension of Theorem 5.1.1 to the Truncated Case

Theorem 5.3.1

Let (X,Y) be a random vector of non-negative, integer-valued components such that

$$P\{X=n\} = P_n \qquad n=k,k+1,\ldots,N, \text{ with } P_n>0 \text{ for } k\leqslant n\leqslant k+\ell$$

where k is non-negative integer, and ℓ is fixed, $1 \leqslant \ell \leqslant N-m$. Suppose that $\{(a_n, b_n)\ n=0,1,\ldots\}$ is a sequence or real vectors with

$$\left.\begin{aligned} &a_n > 0 \quad n=0,1,\ldots,m, \quad a_n = 0,\ n > m \\ &b_n > 0 \quad n=0,1,\ldots,\ell \end{aligned}\right\} \qquad (5.3.1)$$

and let $\{c_n\}$ be defined by $\left\{c_n = \sum_{r=0}^{n} a_r b_{n-r}\ ,\ n=0,1,\ldots,N\right\}$.

If, whenever $P_n > 0$ $n=k,k+1,\ldots,N$

$$P(Y=r|X=n) = \frac{a_r b_{n-r}}{c_n} \qquad r=0,1,\ldots,n\ . \tag{5.3.2}$$

Then

$$P(Y=r|X=Y) = P(Y=r|X=Y+1, Y \geq k) = \ldots = P(Y=r|X=Y+\ell, Y \geq k)$$

$$\text{for a fixed } \ell:\ 1 \leq \ell \leq N-m,\quad r=k,k+1,\ldots,N \tag{5.3.3}$$

iff

$$P_n = \begin{cases} P_k \dfrac{c_n}{c_k} \theta^{n-k} & \text{for some } \theta > 0 \text{ if } n=k,k+1,\ldots,m+\ell \\ d_n, \text{ a constant} & \text{if } m+\ell < n \leq N. \end{cases} \tag{5.3.4}$$

<u>Proof</u> (An outline only is given, since the proof follows on similar lines to the proof of theorem 5.1.1)

"Only if" part of the proof.

From

$$P(Y=r|X=Y) = P(Y=r|X=Y+1, Y \geq k)$$

we find

$$\frac{P_{r+1}}{c_{r+1}} = \frac{P_k}{c_k}\,\theta^{r+1-k} \qquad \text{for } r=k,k+1,\ldots,m. \tag{5.3.5}$$

Also from

$$P(Y=r|X=Y+1, Y \geq k) = P(Y=r|X=Y+2, Y \geq k)$$

and using (5.3.5) we have

$$\frac{P_{m+2}}{C_{m+2}} = \frac{P_k}{C_k} \theta^{m+2-k}. \qquad (5.3.6)$$

Finally from

$$P(Y=r|X=Y+\ell-1, Y \geqslant k) = P(Y=r|X=Y+\ell, Y \geqslant k)$$

and taking into consideration the previous ℓ-1 equations we get

$$\frac{P_n}{C_n} = \frac{P_k}{C_k} \theta^{n-k} \qquad n=k,k+1,\ldots,m+\ell. \qquad (5.3.7)$$

The "if" part of the proof can be obtained using an argument identical to the one used for the "if" part of Theorem 5.1.1.

The following corollary is a direct consequence of Theorem 5.3.1.

<u>Corollary 5.3.1</u> (Characterization of the Truncated Binomial)

Consider the random vector (X,Y) as in Theorem 5.3.1. Suppose that the conditional distribution of Y on X is Hypergeometric as in (5.2.1). Then

$$P(Y=r|X=Y) = P(Y=r|X=Y+1, Y \geqslant k) = \ldots = P(Y=r|X=Y+N-m, Y \geqslant k) \quad (5.3.8)$$

iff P_n is truncated binomial of the form given by (3.6.2).

5.4 Characterizations when the Distribution of Y|X is Truncated.

By making some changes in the conditions on the sequence $\{a_n, b_n\}$ the following theorem can be derived. This gives rise to characterizations of finite distributions, based on truncated conditional distributions.

<u>Theorem 5.4.1</u>

Let (X,Y) be a r.v. as in Theorem 5.3.1. Suppose that $\{a_n\}$ n=k,k+1,... is a sequence of real numbers such that $a_n > 0$ for n=k,k+1,...,m,

$a_n=0$, $m < n \leq N$ and $\{b_n\}$ $n=0,1,2,\ldots$ is another sequence with $b_n > 0$ $n=0,1,\ldots,N-m$ and $b_n=0$ for $N-m < n \leq N$.

Define,

$$c_n = \sum_{r=k}^{n} a_r b_{n-r} \qquad n=k,k+1,\ldots,N.$$

Then, if whenever $P_n > 0$ $n=k,k+1,\ldots,N$

$$P(Y=r|X=n) = \frac{a_r b_{n-r}}{c_n} \qquad r=k,k+1,\ldots,n.$$

Then, condition (5.3.3) is equivalent to (5.3.4).

Proof The proof follows in the same manner as that for Theorem 5.3.1.

As an application of Theorem 5.4.1, we have the following corollary.

Corollary 5.4.1 (Characterization of the Distribution which is the Convolution of a Binomial with a Truncated Binomial).

Let (X,Y) be as in Theorem 5.3.1. Suppose that the conditional distribution Y|X is Hypergeometric, truncated at k-1, i.e.

$$P(Y=r|X=n) = \frac{\binom{m}{r}\binom{N-m}{n-r}}{\sum_{i=k}^{n}\binom{m}{i}\binom{N-m}{n-i}}. \tag{5.4.1}$$

Then, condition (5.3.8) holds iff the distribution of X is the convolution of a Binomial (N-m,p), with a Binomial (m,p) truncated at k-1, i.e.

$$P(X=n) = \frac{p^n q^{N-n} \sum_{r=k}^{n}\binom{m}{r}\binom{N-m}{n-r}}{I_p(k,m-k+1)}. \tag{5.4.2}$$

Proof Consider the following sequences,

$$a_r = \frac{\binom{m}{r} \pi^r \phi^{m-r}}{\sum_{i=k}^{m} \binom{m}{i} \pi^i \phi^{m-i}} \qquad \begin{array}{l} r=k,k+1,\ldots \\ 0 < \pi < 1 \\ \phi=1-\pi \end{array} \tag{5.4.3}$$

(i.e. Binomial truncated at k-1.)

and

$$b_n = \binom{N-m}{n} \pi^n \phi^{N-m-n} \qquad n=0,1,\ldots \tag{5.4.4}$$

Then c_n, $n=k,k+1,\ldots,N$ is given by

$$c_n = \frac{\pi^n \phi^{N-n} \sum_{r=k}^{n} \binom{m}{r}\binom{N-m}{n-r}}{I_p(k,m-k+1)} . \tag{5.4.5}$$

Clearly $a_n > 0$ $n=k,k+1,\ldots,m$ and $b_n > 0$ $n=0,1,\ldots,N-m$. It can be checked easily that the truncated Hypergeometric Distribution (5.4.1) can be decomposed into the form $\frac{a_r b_{n-r}}{c_n}$ $r=k,k+1,\ldots$ with the sequences a_n, b_n, c_n defined as in (5.4.3), (5.4.4), (5.4.5). Hence, applying the result of Theorem 5.4.1 we find that, condition (5.3.8) holds iff

$$P_n = P_k \frac{c_n}{c_k} \theta^{n-k} \qquad \begin{array}{l} n=k,k+1,\ldots,N \\ \text{and some } \theta > 0. \end{array} \tag{5.4.6}$$

Substituting c_n, c_k in (5.4.6) from (5.4.5) and straightforward manipulation shows that P_n can only take the form given in (5.4.2).

Note 1 Corollaries 5.3.1, 5.4.1 can also be considered as special cases of a theorem which is the extension of Theorem (3.6.2) to thé case under study in this chapter. This extension is a generalization of Theorem 5.3.1 and is obtained by considering the sequence $\{a_n : n=0,1,\ldots\}$ to be positive for $k \leqslant n \leqslant m$ instead of defining it to be positive for $0 \leqslant n \leqslant m$. It is also necessary to assume that

$$P(Y=r|X=n) = \frac{a_r b_{n-r}}{c_n} \qquad \begin{array}{l} r=0,1,\ldots,n \\ n=k,k+1,\ldots N. \end{array}$$

<u>Note 2</u> Corollary 5.4.1 can be used to provide the answer to a problem similar to Problem 3.6.1 examined in Chapter 3. This problem arises from the paper by Patil and Seshadri. These authors showed that if Y and Z are independent, and X = Y+Z, then Y|X is Hypergeometric iff Y, Z have Binomial distributions. The question can now be asked as to whether that result can be extended to the truncated case. In other words, is the condition that Y|X has a truncated Hypergeometric distribution necessary and sufficient for Y to be Binomial and Z to be truncated Binomial? As Corollary 5.4.1 shows the "necessary" part is true. But the "sufficient" part is not. The reason is that if Y|X is truncated Hypergeometric and the condition (5.3.8) holds, then from Corollary 5.4.1, X is the convolution of a Binomial with a truncated Binomial. But, using a method similar to the one employed in the Note of Corollary 3.6.1, one can show that the convolution of a Binomial and a truncated Binomial, is not uniquely decomposable into a Binomial and a truncated Binomial. Since independence of Y and Z implies (5.3.8) it is clear that the answer to the problem is in the negative.