APPENDIX

Summarizing Table

In the Tables which follow we give the more important characterizations studied in the thesis. In each entry of the tables the results are given in the following manner.

1st Column: The distribution which is characterized.

2nd Column: Initial assumptions about the form of the distributions required.

3rd Column: Any necessary additional assumptions denoted by $A_1, A_2, \ldots,$ where A_i is specified at the end of this note.

4th Column: The Characterizing Condition.
(N.S. denotes that the condition is necessary and sufficient.)
When a condition has been shown by a counter-example (c.e.) as not being sufficient, it is tabulated with the label $\not{S}$.

5th Column: The reference for each result. The number in the parenthesis following the name of the author denotes the page of the thesis where the result appears. Alternative proofs, where they exist, are denoted by the letter a.
The form of the distributions in the tables and the parameter conditions can be found in Section 1.2 (pp. 7-15).

Notes on the Conditions in Column 3.

$A_1(\theta)$: The parameter θ of the distribution appearing in the second column is variable (in all other cases it is assumed that the parameter(s) of these distributions is(are) fixed.

A_2: $G_X^{(r)}(t)$, $r=1,2,\ldots$ exist.

A_3: $\dfrac{\partial^{n_1+n_2+\ldots+n_s}}{\partial^{n_1} t_1 \ldots \partial^{n_s} t_s} G_X(t)$ $n_i > 0$, $i=1,2,\ldots,s$ exist.

A_4: $P(Y=r|X=n)$ is of the form $\dfrac{a_r b_{n-r}}{c_n}$.

A_5: $\{b_n\}$ is the s-fold convolution of $\{a_n\}$, s fixed.

A_6: $P(Y_1=r_1, Y_2=r_2 | X_1=n_1, X_2=n_2)$ is of the form $\dfrac{a_{r_1,r_2} b_{n_1-r_1, n_2-r_2}}{c_{n_1,n_2}}$

A_7: $P(\underset{\sim}{Y}=\underset{\sim}{r}|\underset{\sim}{X}=\underset{\sim}{n})$ is of the form $\dfrac{a_{\underline{r}} b_{\underline{n}-\underline{r}}}{c_{\underline{n}}}$.

A_8: $F_1(\lambda)$ is absolutely continuous with density

$$f(\lambda|\theta) = e^{-\lambda\theta} \frac{\phi(\lambda)}{\psi(\theta)} \lambda \in (a,b), \quad \phi(\lambda), \psi(\theta) > 0.$$

A_9: The distribution of X is uniquely determined by its factorial moments.

A_{10}: $\int_0^\infty \lambda^x dF_1(\lambda) < \infty$ for $\lambda > 0$, $x=0,1,\ldots$.

<u>Remark</u> It is assumed throughout that in the Binomial distribution $B(r,n,p)$ p is independent of n. It is also assumed that the conditional distribution of $Y|X$ is independent of the parameter(s) of the distribution of X.

Summary Table

Characterization of the Dist. of X	Required form of the Dist. of $(Y\|X=n)$	Additional Assumptions	Characterizing Condition		Reference	
Poisson	Binomial		N.S.	$P(Y=r) = P(Y=r\|X=Y)$		Rao-Rubin (26)
"	"		"	"	a	Shanbhag (27)
"	"		"	"	a	Shanbhag (44)
"	"	$A_1(p)$	"	"		Van-der-Vaart (21)
Neg. Binomial	Neg. Hypergeometric	A_2	"	"		Patil & Ratnaparkhi (47)
"	"		"	"		Panaretos (46)
Trun. Poisson	Binomial		"	$P(Y=r\|Y\geqslant k) = P(Y=r\|X=Y)$		Rao-Rubin (31)
"	"		"	"	a	Panaretos (53)
Trun. Neg. Binomial	Neg. Hypergeometric		"	"		Panaretos (53)
Poisson * Trun. Poisson	Trun. Binomial		"	"		Panaretos (55)
Neg. Binomial * Trun. Neg. Binomial	Trun. Neg. Hypergeom.		"	"		Panaretos (60)

Summary Table (Cont.)

Characterization of the Dist. of X	Required form of the Dist. of $(Y\|X=n)$	Additional Assumptions	Characterizing Condition			Reference
Double Poisson	Double Binomial		N.S.	$P(Y_1=r_1, Y_2=r_2) = P(Y_1=r_1, Y_2=r_2 \| \text{undam.}) = P(Y_1=r_1, Y_2=r_2 \| \text{dam.})$		Talwalker (36)
"	"		"	"	a	Shanbhag (36)
"	"		"	$P(Y_1=r_1, Y_2=r_2) = P(Y_1=r_1, Y_2=r_2 \| X_1=Y_1, X_2=Y_2) = P(Y_1=r_1, Y_2=r_2 \| X_1=Y_1, X_2>Y_2)$		Shanbhag (69)
Double Neg. Binomial	Double Neg. Hypergeom.		"	"		Panaretos (72)
Trun. Double Poisson	Double Binomial		"	$P(Y_1=r_1, Y_2=r_2 \| Y_1\geqslant k_1, Y_2\geqslant k_2) = P(Y_1=r_1, Y_2=r_2 \| X_1=Y_1, X_2=Y_2) = P(Y_1=r_1, Y_2=r_2 \| X_1=Y_1, X_2>Y_2, Y_2\geqslant k_2)$		Panaretos (74)
Trun. Double Neg. Binomial	Double Neg. Hypergeom.		"	"		Panaretos (75)
Double (Poisson * Trun. Poisson)	Trun. Double Binomial			"		Panaretos (77)
Double (Neg. Binomial * Trun. Neg. Binomial)	Trun. Double Neg. Hypergeometric		"	"		Panaretos (77)

Summary Table (Cont.)

Characterization of the Dist. of X	Required form of the Dist. of (Y|X=n)	Additional Assumptions	Characterizing Condition			Reference
Multiple Poisson	Multiple Binomial		N.S.	$P(\underset{\sim}{Y}=\underset{\sim}{r}) = P(\underset{\sim}{Y}=\underset{\sim}{r}\mid\underset{\sim}{X}=\underset{\sim}{Y}) =$ $= P(\underset{\sim}{Y}=\underset{\sim}{r}\mid X^{(j)}>Y^{(j)}), j=2,3,\ldots,s$		Panaretos (90)
Multiple Neg. Binomial	Multiple Neg. Hypergeometric		"	"		Panaretos (91)
Neg. Multinomial	Multivariate Inverse Hypergeometric		"	"		Panaretos (91)
"	"	A_3	"	$P(\underset{\sim}{Y}=\underset{\sim}{r}) = P(\underset{\sim}{Y}=\underset{\sim}{r}\mid\underset{\sim}{X}=\underset{\sim}{Y})$		Patil & Ratnaparkh (93)
Trun. Multiple Poisson	Multiple Binomial		"	$P(\underset{\sim}{Y}=\underset{\sim}{r}\mid\underset{\sim}{Y}\geqslant\underset{\sim}{k}) = P(\underset{\sim}{Y}=\underset{\sim}{r}\mid\underset{\sim}{X}=\underset{\sim}{Y}) =$ $= P(\underset{\sim}{Y}=\underset{\sim}{r}\mid X^{(j)}>Y^{(j)}, Y_j\geqslant k_j), j=2,3,\ldots,s$		Panaretos (94)
Trun. Multiple Neg. Binomial	Multiple Neg. Hypergeometric		"	"		Panaretos (94)
Trun. Neg. Multinomial	Multivariate Inverse Hypergeometric		"	"		Panaretos (95)
Neg. Multinomial * Trun. Neg. Multinomial	Trun. Multivariate Inverse Hypergeometric		"	"		Panaretos (95)

Summary Table (Cont.)

Characterization of the Dist. of X	Required form of the Dist. of (Y|X=n)	Additional Assumptions	Characterizing Condition		Reference	
"Modified" Binomial	Hypergeometric		N.S.	$P(Y=r\|X=Y) = P(Y=r\|X=Y+j)$ $j=1,2,\ldots,\ell,\ \ell$ fixed, $1 \leqslant \ell \leqslant N-m$.		Panaretos (101)
Binomial	Hypergeometric		N.	$P(Y=r) = P(Y=r\|X=Y)$		Patil & Ratnaparkh (103)
"	"		$\mathcal{S}$	"	c.e.	Panaretos (103)
Trun. Binomial	Hypergeometric		N.S.	$P(Y=r\|X=Y) = P(Y=r\|X=Y+j,\ Y \geqslant k)$ $j=1,2,\ldots,N-m$.		Panaretos (107)
Binomial * Trun. Binomial	Trun. Hypergeometric		"	"		Panaretos (108)
Double Binomial	Double Hypergeometric		"	$P(\underset{\sim}{Y}=\underset{\sim}{r}\|\underset{\sim}{X}=\underset{\sim}{Y}) = P(\underset{\sim}{Y}=\underset{\sim}{r}\|\underset{\sim}{X}=\underset{\sim}{Y}+\underset{\sim}{j})$ $j_i=1,\ldots,N_i-m_i,\quad i=1,2.$		Panaretos (123)
Trun. Double Binomial	"		"	$P(\underset{\sim}{Y}=\underset{\sim}{r}\|\underset{\sim}{X}=\underset{\sim}{Y}) = P(\underset{\sim}{Y}=\underset{\sim}{r}\|\underset{\sim}{X}=\underset{\sim}{Y}+\underset{\sim}{j},\underset{\sim}{Y} \geqslant \underset{\sim}{k})$ $j_i=1,2,\ldots,N_i-m_i,\quad i=1,2.$		Panaretos (130)
Double (Binomial * Trun. Binomial)	Trun. Double Hypergeometric		"	"		Panaretos (130)
Multiple Binomial	Multiple Hypergeometric		"	$P(\underset{\sim}{Y}=\underset{\sim}{r}\|\underset{\sim}{X}=\underset{\sim}{Y}) = P(\underset{\sim}{Y}=\underset{\sim}{r}\|X_1=Y_1+R_1,\ldots,X_s=Y_s+R_s)$ $R_i=0,1,\ldots,N_i-m_i,\quad \Sigma R_i \neq 0.$		Panaretos (138)
Multinomial	Multivariate Hypergeometric		"	$P(\underset{\sim}{Y}=\underset{\sim}{r}\|\underset{\sim}{X}=\underset{\sim}{Y}) = P(\underset{\sim}{Y}=\underset{\sim}{r}\|X_1=Y_1+R_1,\ldots,X_s=Y_s+R_s)$ $0 < \Sigma R_i \leqslant n-r.$		Panaretos (139)

Summary Table (Cont.)

Characterization of the Dist. of X	Required form of the Dist. of (Y|X=n)	Additional Assumptions		Characterizing Condition	Reference
Trun. Multiple Binomial	Multiple Hyper-geometric		N.S.	$P(\underset{\sim}{Y}=\underset{\sim}{r} \mid \underset{\sim}{X}=\underset{\sim}{Y})$ $= P(\underset{\sim}{Y}=\underset{\sim}{r} \mid X_1=Y_1+R_1, \ldots, X_s=Y_s+R_s;\ Y_1 \geqslant k_1, \ldots, Y_s \geqslant k_s)$ $R_i=0,1,\ldots,N_i-m_i \quad \Sigma R_i \neq 0$	Panaretos (146)
Trun. Multinomial	Multivariate Hypergeometric		"	$P(\underset{\sim}{Y}=\underset{\sim}{r} \mid \underset{\sim}{X}=\underset{\sim}{Y})$ $= P(\underset{\sim}{Y}=\underset{\sim}{r} \mid X_1=Y_1+R_1, \ldots, X_s=Y_s+R_s;\ Y_1 \geqslant k_1, \ldots, Y_s \geqslant k_s)$ $0 < \Sigma R_i \leqslant n-r$	Panaretos (146)
Multinomial * Trun. Multinomial	Trun. Multivariate Hypergeometric		"	"	Panaretos (146)
Poisson (λ)	Binomial (p) ^ F(p)		"	Y ~ Poisson (λp) ^ F(p)	Panaretos (176)
Poisson	Binomial (p) ^ F(p)		"	$G_Y(t+1) = C^* G_{Y\|X=Y}(t)$	Panaretos (186)
Poisson (λ) ^ $F_1(\lambda)$	Binomial (p) ^ F_2(p)	A_9, A_{10}		Y ~ Poisson (λp) ^ $F_1(\lambda)$ ^ F_2(p)	Panaretos (180)

Summary Table (Cont.)

Characterization of the Dist. of $(Y\mid X=n)$	Required form of the Dist. of X	Additional Assumptions	Characterizing Condition		Reference	
Binomial	Poisson (λ) λ variable		N.S.	$P(Y=r) = P(Y=r\mid X=Y)$		Srivastava & Srivastava (33)
"	Poisson	A_4	"	"		Shanbhag (45)
Neg. Hypergeometric	Neg. Binomial	A_4, A_5	"	"		Panaretos (48)
"Modified" Binomial	Trun. Poisson (λ) λ variable		N.	$P(Y=r\mid Y\geqslant k) = P(Y=r\mid X=Y)$		Srivastava & Singh (34)
"Modified" Binomial	Trun. Poisson		$\not S$	"	c.e.	Panaretos (34)
Trun. Neg. Hypergeometric	Neg. Binomial * Trun. Neg. Binomial	A_4	$\not S$	"	c.e.	Panaretos (62)
Trun. Binomial	Poisson * Trun. Poisson	A_4	$\not S$	"	c.e.	Panaretos (57)
Double Binomial	Double Poisson (λ,μ) λ,μ variables		N.S.	$P(\underset{\sim}{Y}=\underset{\sim}{r}) = P(\underset{\sim}{Y}=\underset{\sim}{r}\mid \text{undam.})$		Srivastava & Srivastava (36)
"	Double Poisson	A_6	"	$P(Y_1=r_1, Y_2=r_2) = P(Y_1=r_1, Y_2=r_2\mid X_1=Y_1, X_2=Y_2)$ $= P(Y_1=r_1, Y_2=r_2\mid X_1=Y_1, X_2>Y_2)$		Panaretos (71)
Multiple Binomial	Multiple Poisson	A_7	"	$P(\underset{\sim}{Y}=\underset{\sim}{r}) = P(\underset{\sim}{Y}=\underset{\sim}{r}\mid \underset{\sim}{X}=\underset{\sim}{Y})$ $= P(\underset{\sim}{Y}=\underset{\sim}{r}\mid X^{(j)}>Y^{(j)})$, $j=2,3,\ldots,s.$		Panaretos (91)

Summary Table (Cont.)

Characterization of the Dist. of $(Y\|X=n)$	Required form of the Dist. of X	Additional Assumptions		Characterizing Condition		Reference
Hypergeometric	Binomial	A_4	N.S.	$P(Y=r\|X=Y) = P(Y=r\|X=Y+j)$, $j=1,2,\ldots,N-m$		Panaretos (102)
Double Hypergeometric	Double Binomial	A_6	"	$P(\underset{\sim}{Y}=\underset{\sim}{r}\|\underset{\sim}{X}=\underset{\sim}{Y}) = P(\underset{\sim}{Y}=\underset{\sim}{r}\|\underset{\sim}{X}=\underset{\sim}{Y}+\underset{\sim}{j})$ $j_i=1,2,\ldots,N_i-m_i$, $i=1,2.$		Panaretos (125)
Multiple Hypergeometric	Multiple Binomial	A_7	"	$P(\underset{\sim}{Y}=\underset{\sim}{r}\|\underset{\sim}{X}=\underset{\sim}{Y}) = P(\underset{\sim}{Y}=\underset{\sim}{r}\|X_1=Y_1+R_1,\ldots,X_s=Y_s+R_s)$ $R_i=0,1,\ldots,N_i-m_i$, $\Sigma R_i \neq 0.$		Panaretos (138)
Multivariate Hypergeometric	Multinomial	A_7	"	$P(\underset{\sim}{Y}=\underset{\sim}{r}\|\underset{\sim}{X}=\underset{\sim}{Y}) = P(\underset{\sim}{Y}=\underset{\sim}{r}\|X_1=Y_1+R_1,\ldots,X_s=Y_s+R_s)$ $0 < \Sigma R_i \leqslant n-r$		Panaretos (142)
Binomial (p) $\wedge$ F(p)	Poisson (λ)		"	$Y \sim$ Poisson (λ) $\wedge$ $F(\lambda)$		Panaretos (181)
Binomial (p) $\wedge$ F_2(p)	Poisson (λ) $\wedge$ $F_1(\lambda)$		$\not S$	$Y \sim$ Poisson (λp) $\wedge$ $F_1(\lambda)$ $\wedge$ $F_2(p)$	c.e.	Panaretos (183)
Binomial (p) $\wedge$ F_2(p)	Poisson (λ) $\wedge$ $F_1(\lambda)$	A_8	N.S.	"		Panaretos (184)