



**ATHENS UNIVERSITY  
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**DEPARTMENT OF STATISTICS**

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**MULTILEVEL MODELS ANALYSIS IN  
HIERARCHICAL DATA STRUCTURE: AN  
APPLICATION TO EDUCATIONAL DATA**

By

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A THESIS

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