CHAPTER 4

4. Modelling the Dynamic Structures of the Greek Stock Market: Applying an ARCH model

In the following section, we estimate a model to examine several issues previously investigated in the economics and financial literature namely a) the relation between the level of market risk and required return, b) the asymmetry between positive and negative returns in their effects on conditional variance, c) fat tails in the conditional distribution of returns d) the contribution of non-trading days to volatility e) the inverse relation between volatility and serial correlation and f) the non-synchronous trading.

We use the model developed by Nelson (1991), assuming an Autoregressive Moving Average representation for $\ln(h_t)$. To allow for the possibility of non-normality in the conditional distribution of returns, we assume that the $z_t \equiv u_t / \sqrt{h_t}$ are i.i.d. draws from the Generalized Error Distribution (GED). The density of a GED random variable normalized to have a mean of zero and a variance of one is given by

$$f(z_t) = \frac{v e^{-2^{-1}\left|\frac{z_t}{\lambda}\right|^v}}{\lambda 2^{v+1/v}\,\Gamma\left(1/v\right)},$$

$-\infty < z < \infty$, $0 < v \leq \infty$, where $\Gamma(.)$ denotes the gamma function, and

$$\lambda \equiv \left(\frac{2^{-2v^{-1}}\Gamma\left(1/v\right)}{\Gamma\left(3/v\right)}\right)^{1/2}.$$

The v is a tail-thickness parameter. When $v = 2$, z has a standard normal distribution. For $v < 2$, the distribution of z has thicker tails than the normal (for $v = 1$, z_t has a double exponential distribution) and for $v > 2$, the distribution of z has thinner tails than the normal (for $v = \infty$, z_t is uniformly distributed on the interval $\left[-\sqrt{3}, \sqrt{3}\right]$).

Thus, we model the log of the conditional variance as:

$$\ln(h_t) = a + \frac{(\Psi_1 L + ... + \Psi_q L^q)}{(1 - \Delta_1 L - ... - \Delta_p L^p)} z_t ,$$

where L is the lag operator.

To account for the contribution of non-trading periods to market variance, we assume that each non-trading day contributes as much to variance as some fixed fraction of a trading day does. If, for example, this fraction is one tenth, than h_t on a typical Monday would be 20 per cent higher than on a typical Tuesday.

Thus we replace the constant term a with:

$$a_t = a_0 + \ln(1 + N_t \delta_0),$$

where N_t is the number of non-trading days between trading days $t-1$ and t, and a_0 and δ_0 are parameters. Fama (1965) and French and Roll (1986) have found that non-trading periods contribute much less than do trading periods to market variance, so we expect that $0 < \delta_0 \leq 1$.

To accommodate the asymmetric relation between stock returns and volatility changes we should use a function $g(z_t)$ instead of z_t. The $g(z_t)$ must be a function of both the magnitude and the sign of z_t. One choice, that in certain cases turns out to give h_t well behaved moments, is to make $g(z_t)$ a linear combination of z_t and $|z_t|$:

$$g(z_t) = \theta z_t + |z_t| - E|z_t|.$$

Over the range $0 < z_t < \infty$, $g(z_t)$ is linear with slope $\theta + 1$, and over the range $-\infty < z_t \leq 0$, $g(z_t)$ is linear with slope $\theta - 1$. Thus, $g(z_t)$ allows the conditional variance process h_t to respond asymmetrically to rises and falls in stock price.

Finally, we model the log of conditional variance as

$$\ln(h_t) = a_t + \frac{(\Psi_1 L + ... + \Psi_q L^q)}{(1 - \Delta_1 L - ... - \Delta_p L^p)} g(z_t).$$

The returns are modeled as:

$$y_t = \mu_0 + \mu_1 h_t + \left(\mu_2 + \mu_3 e^{-h_t / \mu_4} \right) y_{t-1} + u_t ,$$

where the conditional mean and variance of u_t at time t are 0 and h_t respectively and $\mu_0, \mu_1, \mu_2, \mu_3$ and μ_4 are parameters.

The $\mu_2 y_{t-1}$ term allows for the autocorrelation induced by discontinuous trading in the stocks making up an index. The $\mu_1 h_t$ term allows the tradeoff between the expected returns and variance. The $\mu_3 e^{-h_t/\mu_4} y_{t-1}$ term allows for the inverse relation between volatility and serial correlation of returns. As we have already stated, it is difficult to estimate μ_4 in conjunction with μ_3 when using a gradient type of algorithm. For this reason, μ_4 is set to the sample variance of the series,

$$\mu_4 = \frac{\sum_{t=1}^{T} (y_t - \bar{y})^2}{T-1}.$$

In order to maximize the likelihood functions, we use the Eviews 3.1 object LogL. The maximum likelihood parameter estimates were computed using the Marquardt algorithm as the BHHH algorithm fails to converge.

For a given ARMA(p,q) order, the $\{z_t\}_{t=1,T}$ and $\{h_t\}_{t=1,T}$ sequences can be easily derived recursively given the data $\{y_t\}_{t=1,T}$ and the initial values $h_1,\ldots,h_{1+\max(p.q+1)}$. Also, $\ln(h_1),\ldots,\ln(h_{1+\max(p.q+1)})$ were set equal to their unconditional expectations $a_0 + \ln(1 + N_{1t}\delta_0),\ldots, a_0 + \ln(1 + N_{1+\max(p,q+1)}\delta_0)$. This allows us to write the log likelihood as:

$$\mathrm{L_T}(\theta) = \sum_{t=1}^{T} \ln(f(y_t \mid I_{t-1};\theta)) =$$

$$= \sum_{t=1}^{T} \ln\left(\frac{v}{\lambda}\right) - \frac{1}{2}\left|\frac{y_t - \mu_0 - \mu_1 h_t - \left(\mu_2 + \mu_3 e^{-h_t/\mu_4}\right) y_{t-1}}{\sqrt{h_t}\lambda}\right|^{v} - \left(1 + v^{-1}\right)\ln(2) - \ln\left[\Gamma\left(v^{-1}\right)\right] - \frac{1}{2}\ln(h_t).$$

To select the order of the ARMA process for $\ln(h_t)$, we use the Schwarz Criterion (SC) (Schwarz (1978)),

$$SC = (-2l + k\ln(n))n^{-1},$$

where k is the number of estimated parameters, n is the number of observations, and l is the value of the log likelihood function using the k estimated parameters. The model with the lowest SC value is chosen as the most appropriate. Hannan (1980) showed that the SC provides consistent order estimation in the context of linear ARMA models. The asymptotic properties of the SC in the context of ARCH models are unknown. We do not use the Akaike Information Criterion (Akaike (1973)) as it tends to choose the model with the higher number of parameters. The table 4.1 lists the SC values for the various ARMA orders of the model[1].

Table 4.1.
Schwarz criterion for exponential-E-GARCH(p,q) in Mean model.

	MA order(q)				
AR order(p)	0	1	2	3	4
0	-5.33388	-5.40385	-5.47692	-5.5028	-5.51895
1	-5.33227	-5.61135	-5.61126	-5.61382	-5.61454
2	-5.35678	-5.61002	**-5.62066**	-5.60734	-5.61569
3	-5.33149	-5.60973	-5.61867	-5.61603	-5.61303
4	-5.33477	-5.60905	-5.60569	-5.60204	-5.61363

The ARMA(2,2) gives the SC lowest value. Nelson applied a similar model in daily returns for CRSP value weighted market index for July 1962 to December 1987 and selected the ARMA(2,1) order. 4.2 gives the parameters estimates, the estimated standard errors and the t-statistics of the Exponential E-GARCH(2,2) in Mean model:

$$y_t = \mu_0 + \mu_1 h_t + \left(\mu_2 + \mu_3 e^{-h_t/\mu_4} \right) y_{t-1} + u_t$$

$$\ln(h_t) = a_0 + \ln(1 + N_t \delta_0) + \frac{\left(\Psi_1 L + \Psi_2 L^2\right)}{\left(1 - \Delta_1 L - \Delta_2 L^2\right)} \left(\theta \frac{u_t}{\sqrt{h_t}} + \left| \frac{u_t}{\sqrt{h_t}} \right| - E \left| \frac{u_t}{\sqrt{h_t}} \right| \right)$$

[1] The appendix provides the Econometric Views Code constructed to estimate the Exponential E-GARCH(p,q) in Mean models that have been applied.

Table 4.2. **Parameters estimates for** exp-EGARCH(2,2) in Mean model	Coefficient	Std. Error	z-Statistic	Prob.
α_0	-8.0154	0.2537	-31.5996	0.000
δ_0	0.3777	0.0479	7.8887	0.000
θ	0.0289	0.0426	0.6783	0.249
Δ_1	1.7464	0.0490	35.6073	0.000
Δ_2	-0.7488	0.0484	-15.4651	0.000
Ψ_1	0.5387	0.0428	12.5845	0.000
Ψ_2	-0.4979	0.0393	-12.6682	0.000
μ_1	2.2854	0.9667	2.3641	0.009
μ_0	-0.0003	0.0002	-1.3339	0.091
μ_2	0.1105	0.0353	3.1325	0.001
μ_3	0.2121	0.0638	3.3234	0.000
v	1.3908	0.0406	34.2749	0.000

The estimated correlation matrix of the parameter estimates is presented in Table 4.3. These are computed from the inverse of the sum of the outer product of the first derivatives evaluated at the optimum parameter values.

Table 4.3. **Estimated correlation matrix for parameter estimates for** exp-EGARCH(2,2) in Mean model (only lower triangle reported)

	α_0	δ_0	θ	Δ_1	Δ_2	Ψ_1	Ψ_2	μ_1	μ_0	μ_2	μ_3	v
α_0	1											
δ_0	-0.1284	1										
θ	-0.0617	-0.1437	1									
Δ_1	-0.0774	0.0778	-0.0911	1								
Δ_2	0.0821	-0.0756	0.0920	-0.9999	1							
Ψ_1	0.2972	0.0002	-0.0494	-0.4498	0.4504	1						
Ψ_2	-0.2803	-0.0490	0.0747	0.2130	-0.2150	-0.9548	1					
μ_1	-0.0807	-0.0183	0.1054	0.0050	-0.0049	-0.0843	0.0841	1				
μ_0	0.0170	0.0512	0.1833	-0.0343	0.0331	0.0245	-0.0001	-0.5411	1			
μ_2	0.0243	-0.0665	-0.0519	-0.0608	0.0606	0.1136	-0.1028	-0.0141	-0.0495	1		
μ_3	-0.0012	0.0538	0.0250	0.0545	-0.0550	-0.0739	0.0678	-0.0142	0.0807	-0.8585	1	
v	0.0388	0.2607	0.0660	-0.0895	0.0876	-0.1568	0.2211	0.0413	-0.0551	0.0125	-0.0397	1

Let now examine the empirical issues raised in the previous Section.

a) Market Risk and Expected Return: The estimated risk premium is positively correlated with conditional variance, with $\mu_1 = 2.2854$ being statistically significant. This agrees with the significant positive relation between returns and conditional variance found by researchers using GARCH-M models (Chou (1987) and French, Schwert and Stambaugh (1987)), but contracts with the findings of Nelson (1991) who used a similar model and of other researchers not using GARCH models (Pagan and Hong (1988)).

b) The asymmetric relation between returns and changes in volatility, as represented by θ is insignificant. According to the leverage effect, θ should be negative, as we should expect the volatility to rise (fall) when returns surprises are negative (positive). Episodes of high volatility should be associated with market drops. But looking at the plots of the daily conditional standard deviation of returns and the log value of the GI (Figure 4.1), we find that high volatility episodes are associated both with market peaks and drops.

c) Fat Tails. It is well known that the distribution of stock returns has more weight in the tails than the normal distribution (much higher kurtosis than 3), and that a stochastic process is thick tailed if it is conditionally normal with a randomly changing conditional variance (like GARCH processes). In our case the model generates thick tails with both a randomly changing conditional variance h_t and a thick tailed conditional distribution for u_t. The estimated v is approximately 1.39 with a standard error of about 0.04, so the distribution of the z_t is significantly thicker tailed than the normal.

d) The estimated contribution of non-trading days to conditional variance is roughly consistent with the results of French and Roll (1986). The estimated value of δ_0 is about 0.38, which statistically significant, so a non-trading day contributes more than a third as much volatility as a trading day.

e) The inverse relation between volatility and serial correlation for GI daily returns as represented by μ_3 term is statistically significant. Thus the conditional mean is a positive non-linear function of conditional variance.

f) The μ_2 term equals 0.11 shows that the positive non-synchronous trading effect exists in the construction of the GI.

Figure 4.1
The Log Value of the General Index of Athens Stock Exchange and the Daily Conditional Standard Deviation of Returns 3/Aug/87 - 30/Jul/99