

# Chapter 6

## Application

### 6.1 Application with real dataset

As we have already mentioned, we will use the data of the latest Greek Parliamentary Elections which took place on the 22th of September in 1996. 32 parties competed, included single parties, cartels of two or more parties and also independent candidates. The total number of valid votes was 6.738.445. Our purpose is to find out what would have happened, with respect to vote shares, if other systems have been applied to these data.

Five computer programs have been set up for each one of the five electoral systems studied in chapter 3. These computer programs take as an input the valid votes for each party and for each lower district, and give as an output the number of the seats obtained from each party. In fact the input file is an  $m \times n$  matrix of valid votes, where  $m$  is the number of the parties and  $n$  is the number of the lower districts. In our study the number of the lower districts is  $n = 56$ , while the number of the parties that competed in the 1996 elections is  $m = 32$ . In fact 26 of the 32 parties obtained total percentage of valid votes less than 1% each. We found impractical to study each one of them separately, as such a small percentage do not allow to the small political parties to obtain seats in the Parliament. We included Political Spring in our analysis despite its small percentage (2,94%), because it can gain seats when some of the analysed systems are used, while with other it can not (e.g. the 1996 system). The problem was how to treat the 26 small parties: to ignore them completely or to treat them as ‘other’ party. The input  $m \times n$  matrix consists of the total valid votes, for each party and for each

lower district, which means that we have to use the valid votes for *all* the parties which take part in the elections. This is very important because the quota that is used in the primary distribution of seats, in all systems studied, uses the total number of valid votes for all the parties. Thus, the complete exclusion of some parties will lead to wrong results. In order to solve this problem, we treated the 26 small parties as ‘other’, which means that we treat these parties as one party, the 7th party. In that case the input matrix consists of 7 rows. The first 6 correspond to the first 6 parties and the last one to ‘other’. The votes of the 26 parties was summed in each lower district. The sum of the votes, of all these parties, gave a percentage of 2,29%. This is a percentage which might allow to ‘other’ to obtain seats, while in fact no one of them would obtain seats. For this reason the computer programs was updated such that the seventh party never obtain seats.

In order to analyse the electoral systems we have to specify if each one of the competitors is an independent candidate, a single party or a cartel of two or more parties. In the elections studied, the 1996 Parliamentary Elections, all of the six parties under consideration are characterized as single parties.

Each formula also needs the district magnitude  $E[l]$  of each lower district  $l$ , in order to compute the parties seats shares. The district magnitude, for the first 25 lower districts is given below:

$$\begin{array}{llllll}
 E[1] = 4 & E[2] = 3 & E[3] = 3 & E[4] = 3 & E[5] = 4 & E[6] = 14 \\
 E[7] = 7 & E[8] = 7 & E[9] = 3 & E[10] = 3 & E[11] = 5 & E[12] = 4 \\
 E[13] = 4 & E[14] = 2 & E[15] = 5 & E[16] = 2 & E[17] = 1 & E[18] = 5 \\
 E[19] = 3 & E[20] = 2 & E[21] = 2 & E[22] = 8 & E[22] = 8 & E[23] = 6 \\
 E[23] = 6 & E[23] = 6 & E[24] = 5 & E[25] = 5 & E[25] = 5 & E[25] = 5
 \end{array}$$

The secondary distribution of seats is done in  $k$  major districts. The lower districts are aggregated and  $k$  major districts are produced. From 1974 up to 1985 nine major districts was used, while from 1989 up to now thirteen major districts are used. In order to investigate significant differences between these two distinctions we ‘run’ each system twice. One for  $k = 9$  and one for  $k = 13$ . The seats that would have been distributed to parties, for each system,

using 13 major districts, for the 1996 election results is given in Table 9. The seats that would have been distributed to parties using 9a major districts are presented in Table 9b.

**Table 9a:** The seats that would have been distributed to parties ,for each system, and for 13 major districts, for the 1996 election results.

|                    | <b>1st</b>   | <b>2nd</b>   | <b>3rd</b>   | <b>4rth</b>  | <b>5th</b>   | <b>6th</b>   |
|--------------------|--------------|--------------|--------------|--------------|--------------|--------------|
| <b>System</b>      | <b>party</b> | <b>party</b> | <b>party</b> | <b>party</b> | <b>party</b> | <b>party</b> |
| <b>1974</b>        | 152          | 137          | 1            | 4            | 4            | 2            |
| <b>1977 (1981)</b> | 156          | 133          | 1            | 4            | 4            | 2            |
| <b>1985</b>        | 164          | 125          | 1            | 4            | 4            | 2            |
| <b>1989</b>        | 136          | 118          | 11           | 12           | 12           | 11           |
| <b>1993 (1996)</b> | 164          | 106          | 0            | 11           | 10           | 9            |

**Table 9b:** The seats that would have been distributed to parties , for each system, and for 9 major districts, for the 1996 election results.

|                    | <b>1st</b>   | <b>2nd</b>   | <b>3rd</b>   | <b>4rth</b>  | <b>5th</b>   | <b>6th</b>   |
|--------------------|--------------|--------------|--------------|--------------|--------------|--------------|
| <b>System</b>      | <b>party</b> | <b>party</b> | <b>party</b> | <b>party</b> | <b>party</b> | <b>party</b> |
| <b>1974</b>        | 154          | 135          | 1            | 4            | 4            | 2            |
| <b>1977 (1981)</b> | 153          | 136          | 1            | 4            | 4            | 2            |
| <b>1985</b>        | 161          | 127          | 1            | 4            | 5            | 2            |
| <b>1989</b>        | 136          | 117          | 11           | 13           | 12           | 11           |
| <b>1993 (1996)</b> | 160          | 110          | 0            | 11           | 10           | 9            |

We notice that, the allocation of the seats to the parties differs a lot when a different system is applied. As we can notice from the Table 9a the first party takes 152 parliamentary seats

when the 1974 system is used and 164 when the 1993 system is used. Therefore, there is a difference of 12 parliamentary seats, which is a significant number. As we see from Table 9b the first party takes 154 seats with the 1974 system and 160 seats with the 1993 system, when 9 major districts are used. The number of the seats that are distributed to the second party, according to the different systems, varies more than the first one. The second party takes much more less seats with the 1993 system than with any one of the other systems, when we use  $k=9$  or  $k=13$  major districts. The small parties which are the third, the fourth, the fifth, and the sixth take the same number of seats with the first three systems, the 1974, the 1981 and the 1985. The system of the 1889 gives them much more seats in both cases, see Tables 9a and 9b. We also present these results graphically. Figures 4a and 4b summarize the results, with respect to seat shares, for the 'big' parties which are the first two parties. Figures 5a and 5b summarize the results, with respect to seat shares, for the 'small' parties which are the rest parties.

Figure 4a: The seats that would have been obtained by the first two parties, when each one of the studied systems has been applied, and if 13 major districts have been used.

Figure 4b: The seats that would have been obtained by the first two parties, when each one of the studied systems has been applied, and if 9 major districts have been used.

Figure 5a: The seats that would have been obtained by the ‘small’ parties, when each one of the studied systems has been applied, and if 13 major districts have been used.

Figure 5b: The seats that would have been obtained by the ‘small’ parties, when each one of the studied systems has been applied, and if 9 major districts have been used.

Figures 4a and 4b show that in both cases ( $k=9$  and  $k=13$ ) there is an important variation in the number of seats that each one of the first two parties obtains, when the different systems are used. In both cases the first party is enforced with the 1985 and the 1993 system. The second party take less seats as we move from the oldest to the newest systems, in both cases. A first look at the two graphs shows no significant differences between the two cases studied, the one with the 13 major districts and the one with the 9 major districts. This will be studied in detail in the next section.

Analogous results for the ‘small’ parties are shown in Figures 5a and 5b. It is obvious that in both cases the 1989 system is the one that enforces more the ‘small’ parties. We also notice that the third party takes no seats with the 1993 system while with the other systems gains a notable number of seats and especially in the case of the 1989 system. Again a simple look at the two graphs shows no significant differences between the two cases studied, the one with the 13 major districts and the one with the 9 major districts.

Also we notice the first party obtains the majority of the parliamentary seats (more than 150 seats) when the 1974, the 1981, the 1985 and the 1993 system are used. All of them are different forms of the reinforced PR systems. Only the 1989 system gives different results. It does not provide the majority of the parliamentary seats to the first party for both  $k=9$  (Table 9b) and  $k=13$  (Table 9a).

In order to evaluate and compare the electoral systems eight well known measures of disproportionality will be used. We will now present these indices evaluated for the seats distributed to each one of the six parties, according the analysed systems. We will again present the results for both  $k=9$  and  $k=13$ , as we did before with the seat shares.

**Table 10a:** The measures of disproportionality evaluated for the seats obtained by each party, according to the analysed systems (13 major districts), for the 1996 election results.

|                    | <b>1974</b> | <b>1981</b> | <b>1895</b> | <b>1989</b> | <b>1993</b> |
|--------------------|-------------|-------------|-------------|-------------|-------------|
| <b>Rae index</b>   | 0,0519      | 0,0519      | 0,0519      | 0,0154      | 0,0401      |
| <b>LH</b>          | 0,1557      | 0,1557      | 0,1557      | 0,0463      | 0,1203      |
| <b>LSq</b>         | 0,0986      | 0,1006      | 0,1094      | 0,0325      | 0,0997      |
| <b>LH-adjusted</b> | 0,3115      | 0,3115      | 0,3115      | 0,0926      | 0,1873      |
| <b>Lijphart</b>    | 0,0917      | 0,1050      | 0,1317      | 0,0383      | 0,1317      |
| <b>Saint-Lague</b> | 0,1509      | 0,1524      | 0,1608      | 0,0140      | 0,0908      |
| <b>d'Hont</b>      | 0,2210      | 1,2531      | 1,3174      | 1,2455      | 0,3174      |
| <b>Regression</b>  | 1,1841      | 1,1855      | 1,1882      | 1,0580      | 1,2442      |

**Table 10b:** The measures of disproportionality evaluated for the seats obtained by each party, according to the analysed systems (9 major districts), for the 1996 election results.

|                    | <b>1974</b> | <b>1981</b> | <b>1985</b> | <b>1989</b> | <b>1993</b> |
|--------------------|-------------|-------------|-------------|-------------|-------------|
| <b>Rae index</b>   | 0,0519      | 0,0519      | 0,0508      | 0,0143      | 0,356       |
| <b>LH</b>          | 0,1557      | 0,1557      | 0,1524      | 0,0429      | 0,1069      |
| <b>LSq</b>         | 0,0994      | 0,0989      | 0,1041      | 0,0312      | 0,0894      |
| <b>LH-adjusted</b> | 0,3115      | 0,3115      | 0,3048      | 0,0859      | 0,2139      |
| <b>Lijphart</b>    | 0,0983      | 0,0950      | 0,1217      | 0,0383      | 0,1138      |
| <b>Saint-Lague</b> | 0,1514      | 0,1511      | 0,1513      | 0,0121      | 0,0813      |
| <b>d'Hont</b>      | 1,2371      | 1,2290      | 1,2933      | 1,2455      | 1,2853      |
| <b>Regression</b>  | 1,1848      | 1,1844      | 1,1838      | 1,0547      | 1,1228      |

The measures of disproportionality<sup>1</sup>, presented in Table 10a, give us some interesting results:

<sup>1</sup>The interpretation of the results is given in the second part of this Chapter (paragraph 6.2) where the results of 10 additional generated datasets are performed.



a) All indices except d'Hont give almost the same result. The 1989 system seems to be the most proportional. It gives significant smallest values of these indices when it is compared with the other systems. b) The d' Hont index shows that the 1993 is the system which provides the smallest overrepresentation of the most overrepresented party. In other words, the values of the d'Hont index correspond to the most overrepresented party, and the smallest overrepresentation appears in case of 1993. c) After the 1989 system the most proportional is the 1993 system. d) The other three systems seem to give similar results. We notice also that the indices Rae, LH, LH-adj. and Lijphart give the same value for two or more systems. This happens because the computations are based only on one dataset. We will see, in the next section, that this does not happen when we use 10 datasets. e) A regression coefficient greater than 1 shows that all systems favor the big parties. Less biased in favor of large parties is given by the 1989 system and the next less biased in favor of large parties is the 1993 system. The other three systems do not differ with respect to the regression index. However, as we move from older to newer systems (1974, 1981, 1985) more bias is given in favor of large parties. f) We notice no significant differences in the results with respect to seat shares and to indices, when 9 major districts are applied. All these results are illustrated in the following figures. Figure 6a presents the indices for  $k=9$  and Figure 6b presents the indices for  $k=13$ . All the above mentioned results are presented in Kalogirou and Panaretos (1999).

Figure 6a: The eight measures of disproportionality for the five electoral systems, for 13 major districts, and for the 1996 election results

Figure 6b: The eight measures of disproportionality for the five electoral systems, for 9 major districts, and for the 1996 election results

## 6.2 Application with generated data sets

In order to confirm, the results taken from only *one* dataset, the one of the 1996 Greek Parliamentary Elections, we use a large number of generated datasets. In this way we find the allocation of the seats, according to each system when other possible datasets have been considered for the elections. We can see the sensitivity of the systems analysed by noticing the allocation of the seats to each party and the sensitivity of the measures of disproportionality we use.

The way that the new datasets are generated has been described in details in section 4.4. We applied the proposed data generation method for  $c_1 = c_2 = 350$ . We generated values from the Normal distribution  $N(0, 350\Psi_{m,i})$ , for the first party ( $m = 1$ ) and the second party ( $m = 2$ ), for each lower district  $i$ . Lets say these values  $\varepsilon_{m,i}$ . The  $\varepsilon_{m,i}$  differs for each party in each lower district, and it depends on the party's votes in this district ( $\Psi_{m,i}$ ). We have tried different values for  $c_1$  and finally we choose the value of 350, because it is a value that permits different election results, without producing significant differences in the total number of the valid votes, in each lower district, in the generated datasets. This is what we wanted to achieve: datasets based on the initial real dataset, which produce different election results, with respect to seat shares, but not very significant differences with respect to vote shares. This permits us to use, to the generated datasets, the same values for the lower district magnitudes with the real dataset. In fact the computation of the district magnitude is based on the vote totals.<sup>2</sup> Furthermore, we noticed that the value of 350 produces datasets that sometimes permit absolute majority of seats, for the first party, and sometimes not. Thus, it will help us to investigate which systems favor the absolute majority of the first party and which systems not. The parameter  $c_i$  determines the variation in the vote totals in the generated datasets, for each district and for each party. For this reason a smaller variation in vote totals was selected for the smaller parties. For the small parties we choose  $c_3 = 100$ .

It is obvious that the number of the parties is again seven with the seventh party being the

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<sup>2</sup>The district magnitude is computed according to the following procedure: the legal number of the state population is divided by the total number of the parliamentary seats, which is 300. This quotient is called National Divisor. The legal number of the state population consists of all people that are registered to the municipal rolls and not only the people that vote. Then the legal number of the state population is divided by the National Divisor. The new quotient is the district magnitude for each lower district.

‘other’. Furthermore, the number of the lower districts is again 56, and again 9 and 13 major districts are considered.

We have generated 10 datasets which contain the valid votes for each one of the six parties and for each lower district. For each dataset we have computed the number of the seats that each party obtains when each one of the five electoral systems, under consideration, is used. The mean value and the standard deviation of the number of the parties’ seats, for each system, and for all these datasets, are given bellow.

**Table 11:** Mean values and standard deviations (in parenthesis) of the seats, for each party, and for each system, for the generated datasets.

| <b>System</b>      | <b>1st<br/>party</b> | <b>2nd<br/>party</b> | <b>3rd<br/>party</b> | <b>4rth<br/>party</b> | <b>5th<br/>party</b> | <b>6th<br/>party</b> |
|--------------------|----------------------|----------------------|----------------------|-----------------------|----------------------|----------------------|
| <b>1974</b>        | 139.8<br>(4.64)      | 150.6<br>(2.46)      | 0.6<br>(0.5)         | 3.90<br>(0.31)        | 4.10<br>(0.31)       | 2.0<br>(0.0)         |
| <b>1977 (1981)</b> | 139.3<br>(2.11)      | 149.10<br>(2.63)     | 0.7<br>(0.47)        | 4.30<br>(0.47)        | 4.50<br>(0.51)       | 2.10<br>(0.31)       |
| <b>1985</b>        | 130.9<br>(2.45)      | 156.05<br>(3.02)     | 0.7<br>(0.47)        | 4.90<br>(0.64)        | 5.20<br>(0.89)       | 2.10<br>(0.31)       |
| <b>1989</b>        | 121.7<br>(2.25)      | 131.4<br>(5.33)      | 10.75<br>(1.29)      | 12.25<br>(1.62)       | 12<br>(1.62)         | 10.30<br>(1.03)      |
| <b>1993 (1996)</b> | 112.20<br>(3.78)     | 156.20<br>(3.14)     | 1.20<br>(2.46)       | 11.20<br>(0.41)       | 10.20<br>(0.41)      | 9<br>(0.0)           |

In order to find the fairest system we computed the eight measures of disproportionality, for each dataset and for each one of the electoral systems under consideration. Thus, for each one of the electoral systems there are  $8 \times 10 = 80$  values of indices. It is obvious that such a large number of values is difficult to interpret. For this reason we use the mean values, and

the standard deviations of the measures, for the 10 datasets. The results are illustrated in the Table 12.

**Table 12:** Mean values and standard deviations (in parenthesis) of the indices for each system, when 13 major districts are used.

|                    | <b>1974</b>        | <b>1981</b>        | <b>1985</b>         | <b>1989</b>        | <b>1993</b>        |
|--------------------|--------------------|--------------------|---------------------|--------------------|--------------------|
| <b>Rae index</b>   | 0,0524<br>(0,0010) | 0,0515<br>(0,012)  | 0,0502<br>(0,0008)  | 0,0149<br>(0,0033) | 0,0359<br>(0,0027) |
| <b>LH</b>          | 0,1521<br>(0,0180) | 0,1548<br>(0,0035) | 0,1507<br>(0,0026)  | 0,0450<br>(0,0101) | 0,1173<br>(0,0254) |
| <b>LSq</b>         | 0,1008<br>(0,0017) | 0,0983<br>(0,0024) | 0,1023<br>(0,0039)  | 0,0301<br>(0,0057) | 0,0903<br>(0,0064) |
| <b>LH-adjusted</b> | 0,3163<br>(0,0047) | 0,3096<br>(0,0071) | 0,3016<br>(0,0052)  | 0,0895<br>(0,0205) | 0,2159<br>(0,0168) |
| <b>Lijphart</b>    | 0,0992<br>(0,0556) | 0,0929<br>(0,0065) | 0,1182<br>(0,0085)  | 0,0308<br>(0,0068) | 0,1192<br>(0,0084) |
| <b>Saint-Lague</b> | 0,1564<br>(0,0048) | 0,1495<br>(0,0075) | 0,1482<br>(0,0075)  | 0,0154<br>(0,0058) | 0,0780<br>(0,0112) |
| <b>d'Hont</b>      | 1,1447<br>(0,3168) | 1,1296<br>(0,3239) | 1,0916<br>(0,04151) | 1,3016<br>(0,1454) | 1,1940<br>(0,3083) |
| <b>Regression</b>  | 1,1866<br>(0,0028) | 1,1832<br>(0,0044) | 1,1799<br>(0,0033)  | 1,0537<br>(0,0122) | 1,1214<br>(0,0253) |

We can visually see each one of the above indices in the following graphs:

Figure 7.1: The mean value of the Rae index for the 10 datasets, for  $k=13$ .

Figure 7.2: The mean value of the LH index for the 10 datasets, for  $k=13$ .

Figure 7.3: The mean value of the LSQ index for the 10 datasets, for  $k=13$ .

Figure 7.4: The mean value of the LH-adj. index for the 10 datasets, for  $k=13$ .

Figure 7.5: The mean value of the Lijphart index for the 10 datasets, for  $k=13$ .

Figure 7.6: The mean value of the S-L index for the 10 datasets, for  $k=13$ .



Figure 7.7: The mean value of the d Hont index for the 10 datasets, for  $k=13$ .

Figure 7.8: The mean value of the Regression index for the 10 datasets, for  $k=13$ .

In order to find the fairest system we seek for the smallest values of the measures of disproportionality. From Table 12 and also the Figures 7.1 to 7.8 we see that all indices, except d'Hont, take the smallest value in the case of the 1989 system. Furthermore, the above figures indicate that the 1989 system give significant small values for all the indices, except d'Hont, with respect to the other systems. Thus, it is obvious that, according to the above indices, the 1989 system is much more fair than the other electoral systems under consideration. This result agrees with our comment in the section 3.3. It was mentioned that the 1989 system includes a completely different procedure in the secondary distribution of seats, which differentiates it with respect to the other systems. This procedure is much more proportional comparing to the procedure that the other systems use. It uses the remainders of votes and not the total votes. The use of the remaining votes has the result that large parties do not use all their votes in the secondary distribution and this means that they do not use again the votes used in the primary allocation of seats. When all votes are used the quota is much larger than the case where the remaining votes are used. However, larger quota allows over-representation for large parties. That is why this system gives the most proportional results. Furthermore, small parties are enforced more in the 1989 system because parties with at least 1% of votes take at least one seat and parties with at least 2% of votes take at least three seats. But why d'Hont gives different results? This index expresses the over-representation of the most overrepresented party. Some parties, those are mainly the large parties, take more seats with respect to their vote shares. Those are the overrepresented parties. The d'Hont index (as well as Lijphart index) is based only on the most over-represented party and not to all parties, as the other indices do. Consequently, according to the d'Hont index, the 1985 system is the one with the smallest over-representation of the most overrepresented party. We have to note that although the real dataset gave the smallest value of the d'Hont index when the 1993 system was used, in the generated datasets this happens for the 1985 system.

As we have seen all the above indices, except the d'Hont give similar results. The high correlation between the seven indices is also illustrated in the Table 13.

**Table 13:** The correlation between the eight measures of disproportionality.

|                  | Rae    | LH     | LSq    | LH-adj. | Lijph. | S-L    | d'Hont |
|------------------|--------|--------|--------|---------|--------|--------|--------|
| <b>Rae index</b> | 1      |        |        |         |        |        |        |
| <b>LH</b>        | 0,935  | 1      |        |         |        |        |        |
| <b>LSq</b>       | 0,954  | 0,919  | 1      |         |        |        |        |
| <b>LH-adj.</b>   | 0,999  | 0,936  | 0,955  | 1       |        |        |        |
| <b>Lijphart</b>  | 0,768  | 0,770  | 0,917  | 0,766   | 1      |        |        |
| <b>S-L</b>       | 0,986  | 0,922  | 0,913  | 0,986   | 0,702  | 1      |        |
| <b>d'Hont</b>    | -0,231 | -0,221 | -0,235 | -0,229  | -0,205 | -0,220 | 1      |
| <b>Regr.</b>     | 0,9741 | 0,917  | 0,915  | 0,974   | 0,720  | 0,977  | -0,222 |

From the above table we notice that there is a high correlation between all the indices, except the d'Hont. In fact the correlations between Rae, L-H, LSq, LH-adj., S-L and regression are very high, greater than 0,9 and sometimes they reach the 1; see for example, the correlation between the Rae and the LH-adj index. Lijphart index presents small correlation with Rae, LH and LH-adj. However, it remains a significant correlation. The d'Hont presents not only a low correlation with all the indices, but also this correlation is negative. This means that the fair systems, present large over-representation of the most overrepresented party.

After the 1989 system the one that gives the smallest values of almost all indices, except d'Hont and Lijphart, is the 1993 system. Comparing the 1993 system with the 1985 the only difference is the thresholds that the first one uses. In fact they are very important as they give much more different results. As we have already mentioned, the important threshold of the 1993 system does not permit to 'small' parties to gain a seat, in any district, in any distribution. 'Small' parties are the parties with total percentage of votes smaller than 3%. This means that these parties do not take a seat not only in the primary distribution but also in any of the following distributions. On the other hand, the parties with percentage of valid votes greater or equal to 3% of the total valid votes of all the parties, in the entire state, they obtain a minimum number of seats. These seats are eliminated from other parties according to the total number of seats. Thus, this regulation take seats from large parties and allocates them to middle parties. This is illustrated in Table 11. The middle parties, which are the fourth, the fifth and the sixth,

take much more seats when the 1993 system is used with respect to not only the 1985, but also the 1974 and 1981 systems. The fourth party occupies seven more seats, the fifth receives six more seats and the sixth party obtains seven more seats, when the 1993 system is used comparing with the 1974, the 1981 and the 1985 systems. As we have noticed only d'Hont and Lijphart give different results. The index that has been proposed by Lijphart is quite similar with d'Hont index. It also deals with the most overrepresented party. They differ on the way that they define the over-representation. Lijphart uses the difference between the vote and the seat shares, while the d'Hont uses the ratio. That is why these two indices give similar results.

A simple look at the figures 7.1 to 7.8 reveals that the other systems (1974, 1981, 1985) give values, for the indices, which are very close. Comparing the first two systems (1974, 1981) the only difference is on the quota that is used in the primary distribution of the seats. The first one uses the Hare quota while the second uses the Droop quota. As it was mentioned in Chapter 3, Droop give more fair results with respect to Hare quota. This result agrees with our finding when the measures are used. All measures, except Lijphart and Saint Lague give the same result: these systems seems to be more fair as we move from the oldest to the newest. Thus, the 1985 system is more fair than the 1974 and the 1981, while the 1974 system is less fair than the other systems. We have already explained why Droop is more fair than Hare: Hare divides total votes with the available seats, while Droop divides total votes with the available seats plus one. Thus, the value of Hare is greater than the value of Droop, when they are applied to the same number of votes and seats. Given that parties take as many seats as many times quota is contained in the number of votes, quota is covered easier, when droop quota is used. More seats are allocated in the primary distribution, in this case, and less seats are available for the other distributions.

It is important to study the Regression index separately, as it is a measure of the large parties' bias. Regression index greater to one reveals that the system favors the large parties. Table 11 indicates that all systems, under consideration, favor the large parties, and that less biased in favor of large parties is given by the 1989 system. Our results agree with Dimitras (1991) comment, who noted that the 1989 system bias much less in favor of large parties. The next less biased system in favor of large parties is the 1993 system. The other three systems give quite similar values for the regression index. Less biased in favor of large parties is given

as we move to from older to newer systems (1974,1981,1985). All the above comments can be summarized to the following graph.

Figure 8: The mean value for each index, and for each system for  $k=13$

|                    | <b>1974</b>        | <b>1981</b>        | <b>1985</b>        | <b>1989</b>        | <b>1993</b>        |
|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|
| <b>Rae index</b>   | 0.052<br>(0.0009)  | 0.0515<br>(0.0012) | 0.0496<br>(0.0008) | 0.0175<br>(0.0025) | 0.0336<br>(0.0020) |
| <b>LH</b>          | 0.1581<br>(0.0023) | 0.1548<br>(0.0035) | 0.1491<br>(0.0026) | 0.0526<br>(0.0076) | 0.1010<br>(0.0061) |
| <b>LSq</b>         | 0.1002<br>(0.0016) | 0.0980<br>(0.0024) | 0.0992<br>(0.0032) | 0.0376<br>(0.0046) | 0.0843<br>(0.0056) |
| <b>LH-adjusted</b> | 0.3163<br>(0.0047) | 0.0309<br>(0.0071) | 0.2957<br>(0.0088) | 0.1053<br>(0.0152) | 0.2021<br>(0.0123) |
| <b>Lijphart</b>    | 0.0938<br>(0.0068) | 0.0908<br>(0.0064) | 0.1112<br>(0.0078) | 0.0435<br>(0.0053) | 0.1112<br>(0.0087) |
| <b>Saint-Lague</b> | 0.1459<br>(0.0300) | 0.1493<br>(0.0075) | 0.1608<br>(0.0544) | 0.0183<br>(0.0049) | 0.0727<br>(0.0112) |
| <b>d'Hont</b>      | 1.2321<br>(0.0152) | 1.2247<br>(0.0141) | 1.2623<br>(0.0507) | 1.1624<br>(0.0693) | 1.2743<br>(0.0190) |
| <b>Regression</b>  | 1.1864<br>(0.0027) | 1.1828<br>(0.0041) | 1.1779<br>(0.0032) | 1.0679<br>(0.0095) | 1.1141<br>(0.0098) |

**Table 14:** Mean values and standard deviations (in parenthesis) of the indices for each system, when 9 major districts are used.

In Table 14, we present the results for 9 major districts when the 10 generated datasets are used. From the Tables 3 and 5 we see that the use of the nine major districts give quite similar results, on the indices, with the use of the thirteen major districts. This means that the result of an electoral systems is not affected from the choice of the major districts (k=9 or k=13). This result can be confirmed by using Hypothesis Test that the mean of the values of the different measures of disproportionality we analyse are equal, when we take into account 9 and

13 major districts (Two sample T-Test); for more details on hypothesis testing, see, Panaretos (1992). When we applied this test we found that there are no statistical significant differences (at  $\alpha=0.05$ ) between the mean values of the measures of disproportionality for 9 and 13 major districts, when each one of the eight indices is used.

**Table 15:** Two sample t-test, for the major districts, for each index.

|                    | <b>T</b> | <b>p-value</b> |
|--------------------|----------|----------------|
| <b>Rae index</b>   | 0.000670 | 0.979          |
| <b>LH</b>          | 0.009636 | 0.922          |
| <b>LSq</b>         | 0.008075 | 0.929          |
| <b>LH-adjusted</b> | 0.001928 | 0.965          |
| <b>Lijphart</b>    | 0,108441 | 0.743          |
| <b>Saint-Lague</b> | 0.000063 | 0.994          |
| <b>d'Hont</b>      | 1.745972 | 0.189          |
| <b>Regression</b>  | 0.006914 | 0.934          |

Finally, we will present eight graphs presenting the mean values and 95% confidence interval for each one of the eight measures of disproportionality. From the following graphs we notice that the confidence interval for the d'Hont index is quite large, and therefore we can not rely on this index in order to compare the electoral systems. On the other hand the results and the 95% confidence intervals for the other indices are more informative for the comparison of the systems.

Figure 9.1: 95% Confidence Interval for the mean Rae index

Figure 9.2: 95% Confidence Interval for the mean L-H index

Figure 9.3: 95% Confidence Interval for the mean LSq index

Figure 9.4: 95% Confidence Interval for the mean L-H adj. index

Figure 9.5: 95% Confidence Interval for the mean Lijphart index

Figure 9.6: 95% Confidence Interval for the mean S-L index



Figure 9.7: 95% Confidence Interval for the mean d' Hont index

Figure 9.8: 95% Confidence Interval for the mean Regression index

### 6.3 Application with the data of the 1999 Elections for the European Parliament

In this section, we present the results of the 1993 (1996) electoral system when the dataset of the 1999 elections for the European Parliament is used.

The latest elections for the European Parliament took place on the 13th of June of 1999 in all the countries of the European Community. For this reason Greek voters voted for their 25 representatives in the European Parliament. Each one of these representatives belongs to a Greek political party. We suppose that, the Greek citizens vote for the same political parties (representatives) for the election of their representatives in the Greek Parliament (not for the European Parliament). We want to find out the results of this **hypothetical** election, when the current electoral system (for the Greek Parliamentary Elections) is used.

The data have been taken from the Ministry of Internal Affairs. In our analysis we have not included the Greek voters from abroad. It was impossible for us to include them, in the analysis, as we did not know the lower district that each one of them belonged to.

The number of the votes and the respective percentages for the six large parties is given in

the following table.

| <b>Party</b>     | <b>votes</b> | <b>%votes</b><br><b>(% used in the analysis)</b> | <b>%votes</b><br><b>(real percentages)</b> |
|------------------|--------------|--|--|
| PASOK            | 2090762      | 32,69  | 32,92                                      |
| NEW DEMOCRACY    | 2301866      | 35,99  | 36   |
| POLOTICAL SPRING | 146039       | 2,28   | 2,27                                       |
| K.K.E.           | 554915       | 8,67   | 8,66                                       |
| SYNAPSISMOS      | 330589       | 5,16   | 5,16                                       |
| DH.K.KI          | 439712       | 6,87   | 6,84                                       |

**Table 16:** The total number of votes and the percentages of votes used in the analysis, taken from the 1999 elections for the European Parliament in Greece. The last column of the table gives the real percentages of votes.

| <b>Party</b>     | <b>seats</b> | <b>%seats</b> | <b>%votes / %seats</b> |
|------------------|--------------|---------------|------------------------|
| PASOK            | 93           | 93            | 0.94                   |
| NEW DEMOCRACY    | 165          | 165           | 1.52                   |
| POLOTICAL SPRING | 0            | 0             | -                      |
| K.K.E.           | 18           | 6             | 0,69                   |
| SYNAPSISMOS      | 10           | 3,3           | 0,63                   |
| DH.K.KI          | 14           | 4.6           | 0,66                   |

**Table 17:** The seats that would have been distributed to parties, using the 1993 (1996) electoral system, for the 1999 election results. It also includes the ratio of the percentage of seats divided by the percentage of votes.

Table 17 confirm all the comments and the results that we have presented for the 1993 (1996) system. We note that the first party is overrepresented a lot while this does not happen for the second party: PASOK which obtained the 32,69% of the total votes, would have obtained the 31% of the total seats, if National Parliamentary Elections would have been applied. On the other hand the first party, NEW DEMOCRACY, which obtained the 35,99% of the total votes, would have obtained the 55% of the total seats, if National Parliamentary Elections would have been applied. This fact consists the most important disadvantage of this system. Middle parties like K.K.E., SYNASPISMOS and DH.K.KI. are enforced by taking seats from the second party. Finally the party POLITICAL SPRING takes no seats. These results are summarized in the following graph.

**Figure 10:** The percentages of votes and the respective percentages of seats that would have been obtained by the six political parties if the 1996 electoral system has been applied, for the 1999 election results for the European Parliament, in Greece.