Chapter 5

Measures of disproportionality

In order to evaluate the analyzed Greek Parliamentary Electoral systems, we examine the concept of electoral disproportionality, performing some different operational measures that have been proposed in the literature. The measures of disproportionality, we use, are described in detail in Gallagher (1991) and Lijphart (1994). We, also, discuss the advantages and the disadvantages of these measures.

An important political consequence of the electoral systems is the effect on the proportionality or disproportionality of the electoral outcomes. Disproportionality means the deviation of the parties' seats shares from their votes shares. Perfect proportionality is the situation in which every party receives exactly the same share of seats with the share of votes it receives. We will use measures of disproportionality in order to evaluate the Greek electoral systems. These systems can be included in the category of PR systems, which means that they try to minimize the disproportionality and to produce an outcome that is close to perfect proportionality, as possible. It is obvious that, although these systems 'seek' for proportional results, the situation of perfect proportionality is impossible. Some systems achieve more proportional results than other systems. For this purpose, we study the concepts of proportionality and disproportionality. Although, they seem to be simple concepts, the question of finding the best way to measure the proportionality or the disproportionality is much more difficult. All these measures have the same point of departure: they begin by noting the differences between the percentages of seats and the percentages of votes receiving by each alternative (political party or independent candidate). They differ on the way that seat and vote deviations are aggregated. When we use measures of proportionality we seek for large values of the measure, in order to obtain the fairest (the most proportional) system. When we use measures of disproportionality we seek for small values of the measure, in order to obtain the fairest system. In fact, measures of proportionality and disproportionality are two sides of exactly the same coin. We will generally use the term ‘measures of disproportionality’ because the values of all these indices increase when the disproportionality increases. Therefore, the indices of disproportionality that we will present in this chapter are alternative ways of measuring the same phenomenon. However, we have to take into account the fact that although PR systems ‘seek’ for proportional results, the notion proportionality or disproportionality is not always the same for the different electoral systems. That is why Gallagher (1991) say that every method of seat allocation generates its own measure of disproportionality, and that many measures of disproportionality implicitly endorse a method of seat allocation.

The above phenomenon consists the main objection that has been raised to the entire family of the disprportionality measures. However, this is a serious problem only if one focuses on the different outcomes exclusively at the district level. As Cox and Shugart (1991) concede ‘whether national seat totals will be proportional to national vote totals depends on many factors - such as additional seats, thresholds, malapportionment, and the geographical distribution of party support - in addition to the formula used to allocate seats within districts’.

In case of PR systems there are two broad categories of measures of disproportionality, corresponding to the two main types of allocation methods, which we have already discussed. The first category of measures concentrate on the absolute difference between parties’ seats and votes as the Largest Remainders methods do. Methods in the second category focus on the ratio between parties’ seats and votes, just as the Highest Averages methods do. We perform eight measures of disproportionality which have been proposed in the literature. The first five indices (Rae’s index, L-H, LSq, L-H adj., S-L) belong to the first category, the Lijphart’s index and d’ Hont index belong to the second category, while the Regression index, is a satisfactory measure of big parties bias.

5.1 Rae's Index

It is the oldest measure of disproportionality and has been proposed by Rae (1967). It uses the average of the deviations. In fact, it sums the absolute differences between vote percentages (v_i) and seat percentages (s_i) and the outcome is divided by the number of the political parties (n):

$$I = \frac{1}{n} \sum |v_i - s_i|$$

The problem with this index is, that it is sensitive to the presence of small parties. Because of the presence of small parties this index underestimates the real disproportionality measure of the systems. In order to make this characteristic more clear we present an example. Suppose that there are only two political parties. Party A with total percentage of votes equal to 69,96 and Party B with total percentage of votes equal to 30,04. Suppose also that, the total percentage of seat shares is equal to 53 for the first party and the total percentage of seat shares is equal to 25 for the second party. In this case (case (a)) the index proposed by Rae is $I = 7,5$. The existence of one additional small party (case (b)) with total percentage of votes equal to 1 and total percentage of seats in the parliament equal to zero, reduces the Rae's index to $I = 5,3$. Therefore, there is a notable decrease in this index, which means more proportional results, because of just a very small party. Furthermore the existence of additional small parties with small percentage of votes and no seats in the parliament causes an additional decrease of the Rae index. This is the case in Greek parliamentary elections as there are many small parties which do not gain seats in the parliament. Rae's index has the tendency to underestimate the disproportionality of PR systems with many small parties. The difference in the value of the Rae index in cases (a), (b) is quite large, although the real difference between the two cases is due to the existence of only one small political party. Therefore, this index has the tendency to give more proportional results. Rae in order to avoid this problem exclude from the study the small parties. For this purpose, he uses a cutoff point usually 5% of votes. Also, he considers all small parties as 'other' in the election statistics.

This index is trying to measure the total disproportionality per election. As an overall measure of disproportionality it is flawed since a plethora of small parties, each of whose total votes exceeds Rae's cutoff point, will bring down the value to an artificial level.

5.2 Loosemore-Hanby's Index

An index that avoids Rae's index disadvantage was proposed by Loosemore and Hanby (1971). Loosemore Hanby's index has become the most widely used measure of disproportionality. This index (L-H) is given by the sum of the absolute differences between vote percentages (v_i) and seat percentages (s_i), as it happens in the case of Rae's Index I, but now the sum is divided by 2 instead of the number (n) of the political parties. Thus, it is given by

$$D = \frac{1}{2} \sum |v_i - s_i| \,.$$

Mackie and Rose (1982, 1991) subtracted D from 100 and called the result the index of proportionality.

It is obvious that, except from the case of the two parties system ($n_i = 2$), where we take the same result with both measures (Rae and L-H), L-H index gives higher values than the Rae's index.

It is

$$D = \frac{1}{2} |v_i - s_i| = \frac{n}{2}\frac{1}{n} |v_i - s_i| = \frac{n}{2}(\frac{1}{n} |v_i - s_i|) = \frac{n}{2} I \Longrightarrow D = \frac{n}{2} I$$

For $n > 2$, we have that $D > I$. Thus, the Loosemore-Hanby index will always yield higher values than the Rae's Index. In the previous example the difference between the two cases $(a), (b)$ is represented more satisfactory as the index goes up. The advantage of this index is that it does not have to disaggregate the 'other' small parties as in the case of Rae's Index. In contrast with Rae's Index, this index is trying to measure the total disproportionality per party.

L-H index may lead to other paradoxes. Suppose that there are 90 voters, 2 seats and two parties, A and B, each one received 68 and 22 votes respectively. When the L.R.-Hare system is applied the two seats are awarded to party A, because the Hare quota is equal to 45. When the Saint Langue system is applied again the two seats are awarded to party A. In both cases the L-H index is equal to 0,25. Suppose, that a third party C joins the fray and wins 10 uncast votes, which means that the distribution of the seats becomes 68-22-10. When the L.R.-Hare system is applied the first seat is awarded to party A and the second to party B. In case of

Saint Langue the distribution of the seats remain 2-0. L-H index is equal to 0,3 in the case of the L.R.-Hare system and 0,32 in the case of saint Langue system. In this example the L-H index indicates that 2-0 is the least disproportional allocation in case of 90 votes, but 1-1 is least disproportional in case of 100 votes. This index always, by definition, slavishly follows the Largest Remainders method.

Although this method is easy to understand, it is weakened by its vulnerability to paradoxes. These and other doubts have lead to the development of other difference-based indices.

5.3 Least Square Index

Although, there is a good idea behind the Rae's proposal, as its rational is that the vote-seat differences are not on its own enough to convey reliable information of the proportionality of an election outcome, we want to know more about how this sum was reached. Does it derive from many parties each having a small vote-seat difference or from a few parties each having a large difference? This solution was given by Gallagher (1991) with the introduction of the least squares measure. The key feature of this index is that registers a few large deviations much more strongly than a lot of small one's.

In order to make the above problem more clear we present the following example. Consider two elections (a) and (b). In election (a) there are only two parties: the first wins 60% of votes and 64% of seats and the second 40% of votes and 36% of seats. In election (b), there are eight parties: four win 15% of votes and 16% of seats, while the other four win 10% of votes and 9% of seats. According to the Loosemore and Hanby's index these two elections are equally disproportional as in both cases the index is equal to 4. Thus, in this case this index is insensitive to the number of parties. Rae's measure gives the first outcome less proportional $(I = 4)$ with respect to the second outcome $(I = 1)$. In order to take into account the Rae's idea without encountering the above problem Gallagher (1991) offered the following solution: the method of least squares. It is widely used in the social sciences, for example, in fitting a least square regression line to a set of data. A least square index would entail squaring the vote-seat difference for each party, adding these values, dividing the sum by two and taking its square root:

$$LSq = \sqrt{\frac{1}{2} \sum (v_i - s_i)^2}$$

This gives an index which measures disproportionality per election rather than per party and runs from 0 to 100. Another way of thinking about what this index does is that it weights the deviations by their own values, making the larger deviations account for a great deal, than small deviations. In case of only 2 parties this index yields exactly the same values as the Rae and Loosemore-Hanby indices. In other cases it gives a medium value between these two measures. Lijphart (1994) characterizes this index as 'the most faithful reflection of disproportionality of election results'. This fact is described by the Gallagher (1991) as 'a happy medium'. This is clear from the results of the three hypothetical situations (see, Lijphart (1994)) presented in Table 8.

situation	**n**$_i$	**v**$_i$	**s**$_i$	**Index**	
A	1	55	60	**I**	5
	1	45	40	**D**	5
				LSq	5
B	1	50	55	**I**	$1,67$
	1	40	45	**D**	10
	10	1	0	**LSq**	$5,48$
C	5	15	16	**I**	1
	5	5	4	**D**	5
				LSq	$2,24$

Table 8: The Rae (I), the L-H (D) and the Least Square (LSq) index for different values of n_i (number of political parties), v_i (number of total votes), and s_i (number of total seats).

Situation A: existence of only two parties, in this case all of the three indices take the same value. Situation B: existence of many small parties with no seats. In this case Rae's disproportional index is low indicating that the system is proportional and Loosemore-Hanby's index is high indicating that the system is disproportional. Least Square Index takes a value almost in the middle of those two. Analogous results can be seen also in situation C.

5.4 Adjusted Loosemore-Hanby (Grofman's index)

It was suggested by Groffman (1985) and is another effort to find a middle course between Rae's and Loosemore-Hanby's index. The difference from the Rae index is that it divides the total amount of disproportionality by the effective number of parties (N) rather than the real number of parties (n). The effective number of parties weights the parties by their relative sizes and almost always takes a value between 2 and the raw number of parties. It can be calculated either on bases of vote share: $N_v = 1/\sum v_i^2$ or seats shares: $N_s = 1/\sum s_i^2$. In order to clarify the concept of the effective number (N) of parties, we give N in the form: $N = \frac{1}{1-F}$, where F can be based on both parties' vote shares (F_v) and parties' seat shares. F_v is equal to $F_v = 1 - \sum v_i^2$, and F_s is equal to $F_s = 1 - \sum s_i^2$. F_v represents the frequency with which pairs of voters would disagree on their choice of parties if an entire electorate interacted randomly. For more details on the effective number of parties see, Laakso and Taagepera (1979).

Consequently, L-H adj. is given by

$$\begin{aligned}
\frac{1}{N}\sum |v_i - s_i| &= \frac{1}{\frac{1}{1-F}}\sum |v_i - s_i| \\
&= (1-F)\sum |v_i - s_i| \\
&= \langle \begin{array}{l} [1-(1-\sum v_i^2)]\sum |v_i - s_i| \\ [1-(1-\sum s_i^2)]\sum |v_i - s_i| \end{array} \\
&= \langle \begin{array}{l} \sum v_i^2 \sum |v_i - s_i| \\ \sum s_i^2 \sum |v_i - s_i| \end{array}
\end{aligned}$$

Like Rae's index, the L-H adj. index measures the amount of disproportionality per party rather than per election. It is an improvement on Rae's index, but is more complicated to calculate rather than the least square index. Also, it does not have the property of the least square index of 'penalizing' a few large disproportionalities more than a host of small ones.

5.5 Lijphart index

Lijphart (1994), in addition to the above well known indices, introduces in his study another index. He simply uses the largest deviation in an election result as an overall index of disproportionality. This largest deviation comes from the percentage of overrepresentation of the most overrepresented party, and this is usually one of the largest parties. Thus, the index is given by

$$\max |v_i - s_i|$$

Where v_i is the total percentage of votes for the most over-represented party and s_i the total percentage of seats obtained by the most over-represented party. The minimum value of the measure is equal to zero and it happens in case of perfect proportionality of the most overrepresented party. Lijphart (1994) argues that the beauty of this index is that it not only makes good sense but it is also the simplest possible way of measuring disproportionality. The idea come from the following fact: The discrepancy between Rae's and Loosemore-Hanby indices can be alleviated by averaging the vote-seat share differences of the larger parties only. For example, the parties which win more than 5 or 10 percent of the vote. In order to be able to apply this measure in different elections and in both two-party and multi-party systems, he uses the two largest parties. Then he took this line of reasoning one step furthermore, this step was simply to use the largest deviation, in an election result, as the overall index of disproportonality.

5.6 Saint-Lague index

It belongs to the set of indices that focus not on the absolute differences between votes and seats for each party, but on the parties' seats and votes ratio, as the highest averages methods do. The relationship between ratio measures and highest averages electoral formulas can be illustrated by performing the concept of disproportionality that Saint-Lague formula tries to minimize, and it is defined in the following way. For each party calculate the difference $s_i/v_i - TS/TV$, where v_i is the total number of votes of the i^{th} party, s_i is the number of seats received by the i^{th} party, TV, is the total number of votes and TS the total number of seats. Then the difference is squared and the resulting square is weighted by the size of the party. If TV and TS are expressed in percentages, the error term for each party equals the $v_i(\frac{s_i}{v_i} - 1)^2$.

$$\text{It is } v_i(\frac{s_i}{v_i} - 1)^2 = \frac{v_i^2}{v_i}(\frac{s_i}{v_i} - 1)^2 = \frac{1}{v_i}[v_i(\frac{s_i}{v_i} - 1)]^2 = \frac{1}{v_i}(s_i - v_i)^2$$

The index involves simply adding the error terms for each party. Thus, the index is given by

$$\sum \frac{1}{v_i}(s_i - v_i)^2,$$

where v_i is the total percentage of votes for the i^{th} party and s_i the total percentage of seats for the i^{th} party. It differs from the previous indices (Rae, L-H, Lijphart) in that it uses the *relative* difference between parties' shares of seats and votes and not the *absolute* difference. Saint Langue index is a measure whose minimum value is zero, in case of full proportionality, and whose maximum value is infinity, when a party with no votes somehow wins a seat. The open ended nature of the range of this index makes it less easily interpreted.

5.7 D'Hont index

A we have already mentioned in chapter 2, the aim of the d'Hont formula is to keep to a minimum the overrepresentation of the most over-represented party. Consequently, if there was to be a d'Hont index, it would have to be simply this: The seats percentage to votes percentage ratio of the most over-represented party. Thus, it is given by

$$\max(s_i \;/\; v_i),$$

where v_i is the total percentage of votes for the over-represented party and s_i the total percentage of seats obtained by the most over-represented party. The minimum value is 1 in case of the ideal proportionality, and the maximum is the infinity, attained if a party with no votes somehow wins a seat. The disadvantage of this index is that it gives unreliable results in the case where a small party gains some degree of overrepresentation. For example, at Italy's 1983 general election, a small party (Val d'Aost Union) with 0,076 per cent of votes won 0,159 per cent of the parliamentary seats. The d'Hont index is equal to 2,085, a value that yields to this election more disproportionality than the real one. However, in most cases the party in question is the largest one and thus the index gives reliable results. If the most overrepresented party is a small one we can refine the index using a cutoff point, e.g. 5% or 10%.

This index has been used by Taagepera and Laakso (1980) in constructing 'proportionality profiles' of the results produced by the electoral systems. It has also been used by Katz (1984) as a measure of disproportionality.

5.8 Regression index

Cox and Shugart (1991) argued that it should be focus attention on the 'political character' of disproportionality. That is, the extend to which different methods of PR systems favor the large parties over the small ones. However, the development of a satisfactory measure of big parties bias turned out to be very hard. Cox and Shugart (1991) offer an intriguing proposal. They regress the parties' seat percentages on the vote percentages. The slope (b) of the regression line (the regression coefficient) provides a simple index of the big parties bias. Suppose that, the x_i represents the vote percentages and the y_i represents the seat percentages. Then, the regression index is given by

$$b = \frac{\sum x_i \; y_i}{\sum x_i^2}$$

We use a regression line with zero intercept because parties with zero percentages of votes obtain no seats. For more details, on Simple Linear Regression, see, Panaretos (1994). Suppose, for example the regression lines of the following figure.

Figure 3: Regression lines of vote percentages on seat percentages

The regression line with slope equal to one indicates an absence of bias, because the ratio of vote percentages to seat percentages is equal to one, for all parties. This case is represented by the second line, where the ratio of vote percentages to seat percentages is equal to one ($\frac{\%v_i}{\%s_i} = 1$). The first line, where the slope is greater than one ($b > 1$) indicates a systematic advantage for larger parties, as the ratio of seat percentages to vote percentages is greater than one ($\frac{\%s_i}{\%v_i} > 1$), which means seat percentages are greater than vote percentages. Furthermore, as we move from small values of the vote percentages, which represent the small parties, to greater values, which represent larger parties, more seats are given with respect to votes. That is why this line indicates large parties bias. In the same sense the line with slope smaller than one ($b < 1$) indicates a small parties bias.