Chapter 3

Greek electoral systems applied in the Parliamentary elections after the hunte (1974-1999).

3.1 General Description

Greek Electoral systems do not fit either the majoritarian or the PR categories and most of the analysed systems (1874, 1977, 1981, 1985) are referred to as **Reinforced PR** ('enishimeni analogiki')**.** However, these systems can be regarded as sufficiently similar to PR and for this reason Lijphart (1994) includes them in his comparative analysis of all PR systems. All of them are **list** systems. In 'list PR' systems voters may or may not be allowed to express a preference to a particular candidate or candidates within the list. In Greece, in some cases, the purest form of list PR systems, which is the 'closed list', is used. It is pure because voters choose only the party they prefer, making no choice to individual candidates. In that case, each party submits a list of candidates prior to the election. The seats the party wins are distributed in rank of the fixed list. Thus, if there are seven seats to be filled, each party will ordinarily submit a list of seven candidates. If one party wins three seats, it elects the top three candidates of the list. In case of an 'open list' system, voters select a party and then, if they wish, they express a preference to a particular candidate, or candidates, within the list. Vote totals are

translated into parliamentary seats occupied by the deputies. The parliament, in all the cases studied, consists of 300 deputies. The distribution of the 288 seats takes place in three steps: the primary, secondary, and tertiary distribution of seats, with the only exception of the 1989 system. A last step followed is for the allocation of 12 additional seats occupied by the State Deputies.

3.2 Description of the Electoral Systems

This section deals with the study of the Electoral Systems applied in the Greek Parliamentary Elections of 1974, 1977, 1981, 1985, 1989, 1993 and 1996. First of all, a detailed description of each formula is given. Then, each formula is expressed in mathematical relations in the form of pseudo-algorithm. Before the description of the systems some terms must be defined.

Lower districts ('nomoi', or 'elassones eklogikes periferies'). The geographical regions in which the state is divided, for the primary distribution of the seats.

Major districts ('meizones eklogikes periferies'). The geographical regions in which the state is divided, for the secondary distribution of the seats.

Higher districts. The geographical regions in which the state is divided, for the tertiary distribution of the seats.

District magnitude. It is the number of the available seats in the corresponding district.

Quota. As it has already been mentioned, in the case of the Largest Remainders PR systems, the quota is the ratio in which the distribution of the seats is based. This ratio involves the number of the seats and the number of the votes, and it depends on the method that is used (e.g. Hare, Droop) and on the district where it is applied (Lower, Major or Higher). In Greece, the term 'electoral measure' is used

3.2.1 1974 Electoral formula[1]

Primary Distribution of Seats.

The state is divided in 56 lower districts which are almost the same as the 52 geographical-administrative districts ('nomoi') of the country. In fact only the prefecture of Attica is divided

[1] **This electoral system is described, in details, in the Greek Government Gazette (1974).**

in 5 districts and the prefecture of Thessaloniki in two districts, due to overpopulation, while the other remain the same.[2] The seats are distributed, in each lower district, among all alternatives: independent candidates, single parties and cartels of two or more parties. The distribution is done according to the total number of votes. For this purpose, the total number of the valid votes, in each lower district is divided by the district magnitude. The integer part of this ratio is known as **Hare** quota. This quota is applied in each district: the total number of valid votes, for each alternative, in each lower district, is divided by the quota. Parties are given as many seats as they have won quotas. Thus, a party takes as many seats as the integer part of its total votes divided by the quota. In case of an independent candidate, he takes one seat only if his total number of valid votes is greater or equal to the quota. In districts with magnitude equal to one the seat is given to the party with the highest total of votes in this district (relative majority). Any remaining available seats are distributed in the following step.

Secondary Distribution of Seats.

The distribution is carried out in 9 **Major Districts.** Each one of them consists of four, five or even ten[3] lower districts. Only some parties are allowed to obtain seats in this distribution. A Party takes part in the allocation of the seats only if it satisfies the following conditions:

- Single parties with a percentage of total votes greater or equal to 17%.
- Cartels of two parties with a percentage of total votes greater or equal to 25%.
- Cartels of more than two parties with a percentage of total votes greater or equal to 30%.

[2] The 56 lower districts are those of 1) A' disrict of Athens, 2) B' district of Athens, 3) A' district of Peireas, 4) B' district of Peireas, 5) The remaining of Attiki, 6)Biotias, 7) Evias, 8) Fthiotidas, 9) Fokidas, 10) Argolidas, 11) Arkadias, 12) Korinthias, 13) Lakonias, 14) Messinias, 15) Etolias and Akarnanias 16)Ahaias 17) Evritanias, 18) Zakinthou, 19) Ilias, 20) kefallinias, 21) Artas, 22)Thesprotias, 23) Ioanninon, 24) Kerkiras, 25) Lefkadas, 26)Prevezas, 27) Grevenon, 28) Karditsas, 29) Kozanis 30)Larissas, 31) Magnisias, 32) Trikalon, 33) Imathias, 34) Kastorias, 35) kilkis, 36) Pellas, 37) Pierias, 38)Serron, 39) Florinas, 40) Halkidikis, 41) A' disrict of Thessaloniki, 42) B' district of Thessaloniki, 43)Dramas, 44)Evrou, 45) Kavalas, 46)Ksanthis, 47) Rodapis, 48) Dodekanissou, 49) Kikladon, 50) Lesvou, 51)Samou, 52) Hiou, 53) Irakliou, 54) Lasithiou, 55) Rethimnis, 56) Hanion.

[3] The 9 Major districts are those of 1) A' disrict of Athens, B' district of Athens, A' district of Peireas, B' district of Peireas, The remaining of Attiki, Biotias, Evias, Fthiotidas, Fokidas, 2) Argolidas, Arkadias, Korinthias, Lakonias, Messinias, 3) Etolias and Akarnanias, Ahaias, Evritanias, Zakinthou, Ilias, kefallinias, 4) Artas, Thesprotias, Ioanninon, Kerkiras, Lefkadas, Prevezas, 5) Grevenon, Karditsas, Kozanis, Larissas, Magnisias, Trikalon, 6) Imathias, Kastorias, kilkis, Pellas, Pierias, Serron, Florinas, Halkidikis, A' disrict of Thessaloniki, B' district of Thessaloniki, 7)Dramas, Evrou, Kavalas, Ksanthis, Rodapis, 8) Dodekanissou, Kikladon, Lesvou, Samou, Hiou, 9) Irakliou, Lasithiou, Rethimnis, Haniou.

- Independent candidates are not included.

If only one party satisfies the above conditions, a second party (either a single party or a cartel of more than two parties) is allowed to take part in this distribution. It is the party i with the maximum value of the ratio r_i. The ratio r_i, for the party i, is defined as the percentage of its total votes divided by a number which describes the nature of the party. This number takes the value zero for the independent candidates, one for single parties, and λ for cartels of λ parties, (λ=2,3..).

If none of the parties satisfies the above conditions, two parties (either a single party or a cartel of more than two parties) are allowed to take part in this distribution. These two parties have the maximum value of the difference of the real percentage of their votes, in the nation, and the legal admissible percentage for the participation in this distribution given by the law (17% for single parties, 25% for cartels of two parties and 30% for cartels of three or more parties).

The remaining available seats, after the primary distribution, are aggregated in each respective major district. The distribution of the remaining seats in the major districts is done according to a new quota. It is defined as the ratio of the total votes for the parties taking place in this distribution, in each major district, divided by the respective remaining seats. Then the integer part of this ratio is taken. Thus, the Hare quota is used again adjusted to the major districts and to the available seats. Parties are given as many seats as they have won quotas.

So far, it is defined the way that the parties receive the seats in the major districts. We do not only want to know the number of the seats each party wins in the major districts, but also how these seats are distributed to the parties in the lower districts. The electoral system includes the procedure in which the seats of major districts are allocated to the lower districts. This procedure is described below:

Allocation of seats of Major districts to Lower districts

First of all, the number of the remaining available seats of the primary distribution is computed, in each lower district. Then the total number of votes is divided by the respective number of the remaining available seats, in each lower district, for the parties taking place in this distribution. The integer part of this ratio is the Hare quota, which is defined in each lower district. Each party takes as many seats, in a district, as the times the quota is contained in the

party's total valid votes of this district. Simply, the parties are given as many seats as they have won quotas. The remaining undisposed seats are given according to the following procedure. Each available seat, in a lower district, is given to the party with the largest remainder of the quotient which is defined by the party's valid votes divided by the quota of this district.

In the lower districts with magnitude equal to two, if there is only one remaining undisposed seat from the primary distribution, the one and only seat is given to the party that has already taken the first seat in this district, in the primary distribution, only if the total number of valid votes of this party in this district divided by two is greater than the total valid votes of each one of the remaining parties, in this lower district.

If with the above mentioned procedure, some parties occupy more seats than they have to according to the secondary distribution, surplus seats are subtracted. The subtraction is done with the use of the following ratio: The total of valid votes for each party, for each lower district, is divided by the number of seats that has already been allocated to this party from the primary and the secondary distribution. The seats that are in excess, of each party, are subtracted from this lower district where the above ratio is the smallest. Each subtracted seat is added in the same lower district to the party that needs this seat according to the secondary distribution of seats. If there are more than two parties that need this seat, it is given to the party with the highest ratio. In each case of equal total votes between parties, in the allocation or substraction of seats, the selection of the party is done randomly. Seats are not subtracted when they have been allocated a) according to the quota, b)in districts with magnitude equal to two.

In fact this procedure can be omitted, as we are interested in the final result of the elections, because the final result of the system is not affected by this procedure. It is not affected because, this procedure is done in such a way that the total number of seats, for each party in the major districts, is equal to the total number of seats for each party in the respective lower districts.

There is a special case for the lower districts with magnitude equal to two. If only one seat is allocated to a party in the primary distribution also the second seat is given to the same party only if half of its total votes is greater than the total votes of each one of the remaining parties.

Tertiary Distribution of Seats.

The parties which participate in this distribution are those who took part in the secondary distribution. The distribution of the seats is done throughout the state. Hence, the **Higher district** consists of the entire state. The quota is defined as the ratio of the total votes, in the nation, of the parties participating in this distribution, divided by the available number of seats. Then, the integer part of the ratio is taken . Thus, the Hare quota is used, adjusted to the entire state and to the available seats. Parties are given as many seats as they have won quotas. If there are still undisposed seats they are given to the party with the highest percentage of votes in the nation.

Distribution of Seats of State Deputies.

In this distribution of the 12 seats, all the parties that participated in the secondary and the tertiary distribution are allowed to participate. A new quota is used for the distribution of the seats. It is defined as the ratio of the total votes, in the nation, of the parties participating in this distribution, divided by 12. Parties are given as many seats as they have won quotas. Any remaining seats are given to the parties with the *largest remainders of votes*. The remainders are the votes that have not been used for the allocation of seats when the quota is used. For example, if the quota Q is used for the seats distribution, with $Q = V \ / \ S$, the remainder of the votes is equal to $U = V - SQ$.

3.2.2 1977 and 1981 Electoral formula[4]

The national elections of 1977 and 1981 were carried out with the exact same system. The distribution of the seats is exactly the same with the 1974 system except for the primary distribution. **Droop** quota is used for the seats allocation in the lower districts, instead of Hare quota. It is defined as the ratio of the total number of valid votes, in each lower district, for all the alternatives (single parties, cartels of parties, independent candidates) divided by the district magnitude, which is the total number of votes, plus one. Parties are given as many seats as they have won quotas. In Greece, this formula is called 'plus one' ('sin ena').

[4] **This electoral system is described, in details, in the Government Gazette (1981).**

3.2.3 1985 Electoral formula[5]

Primary Distribution of Seats.

The state is divided in 56 lower districts which are almost the same as the 52 geographical-administrative districts of the country. Some of them are divided in more districts due to of overpopulation. e.g. Attica is divided in 5 districts. In fact, the 56 lower districts are exactly the same as the previous systems. The seats are distributed, in each lower district, among all alternatives: independent candidates, single parties and cartels of two or more parties. The distribution is done according to the total number of votes. For this purpose, the distribution of seats for the single parties and the cartels of more than two parties is done in the following way: In districts with magnitude equal to one, and those are the districts with only one available seat, the one and only seat is given by the use of the *plurality rule*. It is also called *relative majority* or *first past the post.* According to this rule the seat is given to the party with the most valid votes, whether or not that party has an absolute majority (50 percent plus one) of the votes cast. In case that more than one parties have the same total number of valid votes, the seat is given to one of them randomly.

In districts with magnitude greater or equal to two, and those are the multi-member districts, seats are given according to *Droop* quota. In each lower district Droop quota is computed. That is, the total number of the valid votes, in each district, divided by the district magnitude increased by one ('sin ena'). Each party occupies as many seats, in a district, as the times the quota is contained in the party's total valid votes of this district. Simply, the parties are given as many seats as they have won quotas.

The distribution of the seats for independent candidates is done in the following way: An independent candidate takes one seat in a lower district only if its total valid votes, in this district, is greater or equal to the droop quota.

If the previous procedure gives in some districts more seats than available, the seats that are in excess are subtracted according to the smallest remainders. The remainders are the seats that have not been used for the allocation of the seats. In a district the surplus seat is subtracted from the party of the smallest remainder. It is the remainder of the division of

[5]**The Greek Parliament vote this electoral system the January of 1985. See, Government Gazette (1985).**

its total valid votes by the district's magnitude. In case two or more parties have the same remainder, the selection of the party is done randomly. Any remaining seats are distributed in the following step.

Secondary Distribution of Seats.

The distribution is carried out in 9 **Major Districts.** Each one of them comes from the aggregation of four, five or even ten lower districts. Single parties and cartels of more than two parties take part in the allocation of the seats, while independent candidates are excluded.

The distribution of the remaining seats of the primary distribution is done in each major district in the following way: The available seats from the primary distribution are aggregated in each respective major district. A new quota is defined in each major district. It is the ratio of the total votes, for the parties taking place in this distribution, divided by the respective remaining seats in each major district (Hare). Each party takes as many seats in a district as the times the quota is contained in the party's total valid votes of this district. Simply, parties are given as many seats as they have won quotas.

The above procedure gives the number of seats that are allocated to each party according to the secondary distribution of seats. In this procedure the seats are allocated to major districts. Thus, the number of the seats that each party obtains is known, in each major district. The remaining seats are allocated to parties in the following stage, the tertiary distribution of seats. For the realization of this step, the secondary distribution of seats in the lower districts, is needed. In simple words we have to know not only the number of the seats obtained by each party in the major districts, but also the number of the seats obtained by each party in the lower districts. For this purpose I perform the procedure for the allocation of seats of major districts to lower districts.

Allocation of seats of Major districts to Lower districts

First of all, the number of the remaining available seats of the primary distribution is computed, in each lower district. Then the total number of votes, in each lower district, for the parties taking place in this distribution, is divided by the respective number of the remaining available seats. The integer part of this ratio is the Hare quota which now is defined in each lower district. Each party takes as many seats, in a district, as many times the quota is contained in the party's total valid votes of this district. Simply, parties are given as many seats as they have

won quotas. The remaining undisposed seats are given according to the following procedure. Each available seat, in a lower district, is given to the party with the largest remainder of the quotient which is defined by the party's valid votes divided by the quota of this district.

In the lower districts with magnitude equal to two, if there is only one remaining undisposed seat from the primary distribution, the one and only seat is given to the party that has already occupied the first seat in this district, in the primary distribution, only if the total number of valid votes of this party in this district divided by two is greater than the total valid votes of each one of the remaining parties in this lower district.

If with the above procedure, some parties occupy more seats than they have to according to the secondary distribution, surplus seats are subtracted.. The subtraction is done with the use of the following ratio: Total valid votes of each party, for each lower district, is divided by the number of seats that has already been allocated to this party from the primary and the secondary distribution. Seats that are in excess, of each party, are subtracted from this lower district where the above ratio is the smallest. Each subtracted seat is added in the same lower district to the party that needs this seat according to the secondary distribution of seats. If there are more than two parties that need this seat, it is given to the party with the largest ratio. In each case of equal total votes between parties, in the allocation or substraction of seats, the selection of the party is done randomly. The seats are not subtacted when they have been allocated a) according to the quota, b)in districts with magnitude equal to two.

In fact this procedure can be omitted, when we are interested in the final result of the elections, because the final result of the system is not affected by this procedure. It is not affected because this procedure is done in such a way that the total number of seats, for each party in the major districts, is equal to the total number of seats for each party in the respective lower districts.

There is a special case for the lower districts with magnitude equal to two. If only one seat is allocated to a party in the primary distribution also the second seat is given to the same party only if half of its total votes is greater than the total votes of each one of the remaining parties.

Tertiary Distribution of Seats.

The parties which participate in this distribution are also the same who participated in

the secondary distribution. Thus, independent candidates are excluded. This distribution is done throughout the state which means that the **Higher district** consists of the entire state. The remaining undisposed seats of the primary and the secondary distribution is computed in each lower district. The remaining seats, in each lower district, are given to the party that has also the *plurality* (relative majority) of its total votes in this lower district only if this party has the *plurality* of the total valid votes throughout the state. For the remaining seats the ratio of the total votes in the nation of the parties participating in this distribution divided by the remaining number of seats, is computed. Parties are given as many seats as the have won quotas[6]. The remaining seats are given to the party that has the *plurality* of total votes throughout the state. Any remaining seats are given to the party with the highest percentage of votes in the nation.

Distribution of Seats of State Deputies.

The parties which participate in this distribution are those who took part in the secondary and also the tertiary distribution. Thus, again ndependent candidates are excluded. The quota is defined as the ratio of the total votes in the nation of the parties participating in this distribution divided by 12 (Hare quota). Parties are given as many seats as the have won quotas. For the remaining seats the following procedure is followed. Let s_i be the number of seats allocated to i with the use of the quota. Any remaining seats are given to the parties according to the following ratio: total valid votes of each party is divided by $s_i + 1$. The first remaining undisposed seat is given to the party with the largest ratio $v_i/(s_i + 1)$. The second is given to the party with the next largest ratio, where s_i are the total seats that the party has already gained. The procedure continues until all remaining seats are given to the parties. (d' Hont formula)

[6]The electoral system includes the procedure with which those seats are allocated to lower districts. This procedure can be excluded as the next stage is the distribution of seats throughout the state and thus the number of seats allocated to each party, in each lower district, is not needed. Furthermore, this procedure does not affect the final result of the electoral formula.

3.2.4 1989 Electoral formula[7]

Primary Distribution of Seats.

The state is divided in 56 lower districts which are almost the same as the 52 geographical-administrative districts, but some of them are divided in more districts due to overpopulation. The seats are distributed in each lower district among all alternatives: independent candidates, single parties and cartels of two or more parties. The distribution is done according to the total number of votes. For this purpose, the distribution of seats for single parties and cartels of more than two parties is done in the following way: In districts with magnitude equal to one, and those are the districts with only one available seat, the one and only seat is given by the use of the *plurality rule*. According to this rule the seat is given to the party with the most valid votes, whether or not that party has an absolute majority (50 percent plus one) of the votes cast. In case there are more than one parties that have the same total number of valid votes, the seat is given to one of them randomly.

In districts with magnitude greater or equal to two, and those are the multi-member districts, seats are given according to *Droop* quota. In each lower district Droop quota is computed. That is the total number of the valid votes, in each district, divided by the district magnitude increased by one ('sin ena'). The total number of valid votes for each party, for each district, is divided by the quota of the district. Each party occupies as many seats in a district as many times the quota is contained in the party's total valid votes of this district. Simply, the parties are given as many seats as they have won quotas.

The distribution of the seats for independent candidates is done in the following way: An independent candidate occupies one seat in a lower district only if its total valid votes in this district is greater or equal to the droops quota of the district.

If the previous procedure offers, in some districts, more seats than available, seats in excess are subtracted according to the smallest remainders. The remainders are the seats that have not been used for the allocation of the seats. In a district the 'surplus' seat is subtracted from the party with the smallest remainder. It is the remainder of the division of its total valid votes by the district's magnitude. In case two or more parties have the same remainder, the selection

[7] **The Greek Parliament vote this electoral system on the 31th of March on 1989. See, Government Gazette (1989).**

of the party is done randomly. Any remaining seats are distributed in the following step.

Secondary Distribution of Seats.

The distribution is carried out in 13 **Major Districts** [8]**.** Each one of them consists of two, three or even six lower districts. Single parties and cartels of more than two parties are allowed to take part in the allocation of the seats, in this stage, while independent candidates are excluded.

The distribution of the remaining available seats is done, in each major district, in the following way: The remaining valid votes for single parties and cartels of more than two paries, are aggregated, in each major district. The remaining votes (*remainder of votes*), for each party, are its votes, that have not been accounted for the seats allocation in the primary distribution. For example, if the quota Q is used for the seats allocation, in the primary distribution of seats, with $Q = V \ / \ S$, the remainder of the votes is equal to $U = V - SQ$. The sum of the remaining votes is divided by the respective available seats, in each major district. Then the integer part of the ratio is taken. Each party occupies, in a district, as many seats as the times that the ratio is contained in the party's total remaining valid votes of this district. If there are still available seats, they are given to the parties (single parties and cartels of more than two paries) according to their remaining valid votes from the above allocation and the one of the first distribution. This means that the party with the largest value of the remaining votes, which have not been accounted for the seats allocation, in the first step of this procedure, takes the first seat. The party with the second largest value occupies the second seat, and so on.

If there are single parties or cartels of more than two parties with a total percentage of valid votes greater or equal to 2%, they obtain at least 3 seats. In case of parties with a total percentage of valid votes smaller or equal to 2%, but not smaller than 1%, they obtain at least one seat. For that purpose, if there are single parties or cartels of more than two parties that have the right to obtain 3 seats and have not reached this number, the next procedure follows. The distribution of seats to these parties is done according to the total percentages of votes, in

[8] The 13 Major districts are those of 1) A' disrict of Athens, B' district of Athens, A' district of Peireas, B' district of Peireas, The remaining of Attiki, 2) Biotias, Evias, Fthiotidas, Fokidas and Evritanias 3) Argolidas, Arkadias, Korinthias, Lakonias, Messinias, 4) Etolias and Akarnanias, Ahaias and Ilias 5) Zakinthou, , Kefallinias, Kerkiras, and Lefkadas,6) Artas, Thesprotias, Ioanninon, Prevezas, 7) Grevenon, Kastorias, Kozanis, Florinas 8)Karditsas, ,Larissas, Magnisias, Trikalon, 9) Imathias, , kilkis, Pellas, Pierias, Serron, , Halkidikis, A' disrict of Thessaloniki, B' district of Thessaloniki, 10)Dramas, Evrou, Kavalas, Ksanthis, Rodopis, 11) Dodekanissou and Kikladon 12) Lesvou, Samou, Hiou, 13) Irakliou, Lasithiou, Rethimnis, Haniou.

major districts, such as: the party with the highest percentage of valid votes, in the district, obtains the first seat, the party with the second highest percentage of valid votes takes the second seat and so on. The procedure continues until they reach the number of three seats. This procedure is not applied to the parties that have already won one or two seats in the primary distribution, in the major districts. If there are single parties or cartels of more than two parties that have the right to obtain one seat and they have not obtained it yet, they take the seat in the major district, where they have gained the higher number of valid votes. If, there is a major district in which there are more parties, that have the right to obtain a seat, than seats, the available seats are given to parties according to the highest percentages of valid votes. The remaining parties obtain the seats that they have to, in the other major districts according to the largest percentages of valid votes.

If there are still available seats, they are distributed to the rest of the parties (single parties or cartels of more than two parties). The same procedure is followed as the one used for the allocation of seats in the secondary distribution. Thus, the sum of the remaining votes (remainders of votes) is divided by the number of the respective available seats, in each major district. The seats that have already been awarded to parties in the primary and the secondary distribution, so far, are not taken into account. Then the integer part of the ratio is taken. Each party occupies, in a district, as many seats as the times the ratio is contained in the party's total remaining valid votes of this district. The remaining seats are given to parties (single parties and cartels of more than two paries) according to their remaining valid votes.

So far, the allocation of seats, in the secondary distribution, is done to the parties in the major districts. Simply, so far, we know the number of seats, each party obtains, in each major district. The system also includes the procedure in which the allocation of the seats is done in the lower districts. This procedure will be omitted, because we interested in the final result of the system and not in the results of each district .

Distribution of Seats of State Deputies.

The parties which participate in this distribution are also those who took part in the secondary and the tertiary distribution. Thus, independent candidates are excluded. The quota is defined as the ratio of the total votes in the nation of the parties participating in this distribution divided by 12 (Hare quota). Parties are given as many seats as the have won quotas. For

the remaining seats the following procedure is followed. Let s_i be the number of seats allocated to the party i with the use of the quota. Any remaining seats are given to the parties according to the following ratio: total valid votes of each party is divided by $s_i + 1$. The first remaining undisposed seat is given to the party with the largest ratio $v_i/(s_i + 1)$,where s_i are the total seats that the party has already gained. The second seat is given to the party with the next highest ratio. The procedure continues until all the remaining seats are given to parties. (d' Hont formula)

3.2.5 1993 and 1996 Electoral formula[9]

Primary Distribution of Seats.

The state is divided in 56 lower districts which are the same as the lower districts used in the previous mentioned systems. The seats are distributed in each lower district among all alternatives: independent candidates, single parties and cartels of two or more parties. The distribution is done according to the total number of votes. For this purpose, the distribution of seats for single parties and cartels of more than two parties is done in the following way: In districts with magnitude equal to one, those are the districts with only one available seat, the one and only seat is given by the use of the *plurality rule* (*Relative majority*, *First past the post).* According to this rule the seat is given to the party with the most valid votes, whether or not that party has an absolute majority (50 percent plus one) of the votes cast. In case more than one parties have the same total number of valid votes, the seat is given to one of them randomly.

In districts with magnitude greater or equal to two, and those are the multi-member districts, seats are given according to *Droop* quota. In each lower district Droop quota is computed. That is, the total number of the valid votes, in each district, divided by the district magnitude increased by one ('sin ena'). The total number of valid votes for each party for each district is divided by the quota of the district. Each party occupies as many seats in a district as many times the quota is contained in the party's total valid votes of this district. Simply, the parties are given as many seats as they have won quotas.

The distribution of the seats for independent candidates is done in the following way: An

[9] **This electoral system is described, in details, in the Government Gazette (1993).**

independent candidate occupies one seat in a lower district only if its total valid votes in this district is greater or equal to the droops quota of the district.

If the previous procedure gives, in some districts, more seats than the available, seats in excess are subtracted according to the smallest remainders. The remainders are the seats that have not been used for the allocation of the seats. In a district the 'surplus' seat is subtracted from the party with the smallest remainder. It is the remainder of the division of its total valid votes by the district's magnitude. In case two or more parties have the same remainder, the selection of the party is done randomly.

If there are parties (single parties, cartels of two or more parties or even independent candidates) with a percentage of valid votes smaller than the 3% of the total valid votes of all the parties throughout the state, then these parties, which might be single parties, cartels of two or more parties or even independent candidates, they are not allowed to gain a seat, in any district, in any distribution. This means that these parties do not occupy a seat; not only in the primary distribution but also in none of the following distributions. On the other hand, the parties with a percentage of valid votes greater or equal to 3% of the total valid votes of all the parties, throughout the state, they obtain a minimum number of seats. This number is the integer part of the 70% of the seats that correspond to the percentage of the party's valid votes, multiplied by 300. If there are parties that have obtained less seats than the number they are entitled to gain, then they are conferred the appropriate number of seats. These seats are taken from other parties according to the total number of seats: the first seat is removed from the party with the smallest percentage of seats, the second seat from the next smallest, with respect to seats, party and so on. If there are two or more parties that are entitled to take seats, then the distribution is done according to the highest percentages of votes. The first seat is given to the party with the largest percentage of valid votes, the second seat to the next largest party, with respect to votes, and so on. When a party gains a seat with the above procedure, it gains the seat in the district with the highest remainders of votes, with respect to the primary distribution of seats. The seats are not allocated to the parties, in the lower districts, where they have occupied a seat in the secondary distribution. In this case the entitled seat is allocated to the lower district with the next largest remainder. Any remaining seats are distributed in the following step.

Secondary Distribution of Seats.

The distribution is carried out in 13 **major districts,** which are exactly the same as the major districts of the previous mentioned system. As it was mentioned in the previous systems each one of these districts comes from the aggregation of two, three or even six lower districts. In this stage, single parties and cartels of more than two parties take part in the allocation of the seats, while independent candidates are excluded.

The distribution of the remaining seats is done, in each major district, according to the total votes: The total valid votes for single parties and cartels of more than two parties, are aggregated, in each major district. The sum of the total votes is divided by the respective available seats, in each major district. Then the integer part of the ratio is taken. Each party occupies, in a district, as many seats as the times the ratio is contained in the party's total valid votes of this district. The exact same procedure, as in the previous systems, is also used for the allocation of the seats in the lower districts.

Tertiary Distribution of Seats.

The parties which participate in this distribution are those who also participated in the secondary and tertiary distribution. Thus independent candidates are excluded. This distribution is done throughout the state and consequently, the **higher district** consists of the entire state. The remaining indisposed seats from the primary and the secondary distribution is computed by using the following quota: it is defined as the ratio of the total votes, in the nation, of the parties participating in this distribution divided by the remaining number of seats. Then, the integer part of the ratio, is taken. Thus, the Hare quota is used, adjusted to the entire state and to the remaining seats. The parties are given as many seats as the have won quotas. The procedure for the distribution of these seats in the lower districts follows. However, it can be omitted from our study, as it is not required neither for the following stages of the distribution, nor does it affect the final result. The indisposed seats, after the tertiary distribution, are awarded to the single party with the highest value of total valid votes throughout the state. There is a special case where these seats are allocated to a cartel of λ parties. This is when there is a cartel with an average percentage of votes for the λ parties greater than the votes percentage of the largest single party.

Distribution of Seats of State Deputies.

The parties which participate in this distribution are those who also participated in the secondary and tertiary distribution. Thus, independent candidates are excluded. Quota is defined as the ratio of the total votes in the nation of the parties participating in this distribution divided by 12 (Hare quota). Parties are given as many seats as the have won quotas. For the remaining seats the next procedure is followed. Let s_i be the number of seats allocated to party i with the use of the quota. Any remaining seats are given to the parties according to the following ratio: total valid votes for each party is divided by $s_i + 1$. The first remaining undisposed seat is given to the party with the highest ratio $v_i/(s_i + 1)$. The next seat is given to the party with the next highest ratio, where s_i is the total seats that the party has already gained. The procedure continues until all the remaining seats are given to the parties. (d' Hont formula).

3.2.6 Comments on the systems

After the description of the electoral systems we can summarize the following: The distribution of the 288 seats is done in three stages. The only exception is the 1989 system, in which the 288 seats were distributed in the primary and the secondary distribution. The 12 additional seats, obtained by the state deputies, are distributed by using a separate procedure. State deputies' seats were introduced in 1974 and were applied in all parliamentary electoral systems up to now. The first distribution is done in 56 lower districts, which are the same in all the cases studied. Although the lower districts remain the same, it does not happen the same to the lower districts magnitude, and consequently to the major districts magnitude. The number of the available seats, in each district, sometimes changes, due to population shifts. From 1926 up to 1985 the available seats in the major district of Athens doubled, while the available seats of the Agaio district has decreased by 1/3. For more details see Pantellis and Triantafillou (1985).

First district: in all systems, all parties, (including single parties, cartels and also independent candidates) are entitled to participate, which means that all of them are able to gain the seats. In fact, it is the only distribution where independent candidates are allowed to participate. Furthermore, each one of the independent candidates, is allowed to obtain at the very most one seat. In most cases, they do not obtain the number of votes that enables them to enter

the parliament. We have to point out that in single-member districts, in all systems studied, the plurality rule is applied. The plurality (relative majority) rule in a single member district is the same as the application of PR d' Hont, for this situation: the party with the highest value of $v_i/(s_i+1)$, with s_i the number of seats received *so far* by the i^{th} party. However, in a single member district s_i is equal to 0 for all competitive parties and hence the seat is received by the party with the highest votes percentage. The application of a quota like the Hare, in single member districts is almost impossible. For example, Hare quota is a single member district, would be Hare $= v/s = v/1 = v$. Thus, a party has to attract all voters, in order to obtain the one and only available seat. One might say that the small parties are favored by this regulation, as it gives them the opportunity to obtain seats, provided that they will achieve a relative majority. What happens in reality is that only large parties achieve this relative majority and especially the first party. In multi-member districts the Hare quota was applied only in 1974, while in all other cases the Droop quota was used. Hare divides total votes by the available seats, while Droop divides total votes by the available seats plus one. Thus, the value of Hare is greater than the value of Droop, when they are applied in the same number of votes and seats. Given that parties occupy as many seats as many times the quota is contained in the number of votes, quota is more easily covered from the parties (large or small), in case of droops. In this case more seats are allocated in the primary distribution, so the remaining seats are less for the other stages. The application of Droop instead of Hare is more important in lower-member districts. In the case of a district with a large number of seats the additional 'one' in the quota, does not affect the quota a lot. In multi-member districts both v and s are large (v = total votes, s =the available seats) and thus the two ratios v/s and $v/s+1$ do not differ a lot. The difference between the two ratios increases for small values of v and s. The systems after the 1985 include a procedure for the elimination of the 'surplus' seats. 'Surplus' seats appear when the distributed seats are more than the available. The procedure is based on the remainders of votes. The votes that have not been used in the allocation of the seats. It is a very rare case and it might happen only, in the case of Droop quota.

Secondary distribution: In all cases, independent candidates are excluded from this distribution. In the 1985, 1989 and 1993 systems all parties and cartels of parties were allowed to participate in the secondary distribution. Since all parties are allowed to participate, the quota

is quite high and only the big parties manage to obtain seats. It is not obvious, but in fact, this regulation excludes from the secondary distribution all parties except the two largest. The previous systems imposed the threshold of 17% for single parties, 25% for cartels of two parties and 30% for cartels of more than two parties. Taking into account that Greece is a two-party system (there are two large parties), this threshold does not matter so much. Pantellis (1988) points out that the abolition of the 17% threshold in 1989 is only of phycological importance. The two large parties participate in this distribution. Furthermore, the participation of all parties in all distributions have a bias towards the second party and favors the first and the small parties; see Dimitras (1991). The distribution is done by using the Hare quota, in each major district. In the case of 1989 the procedure differs. It uses the remaining votes of the primary distribution. The remaining votes for each party are its votes that have not been accounted for the seats allocation, in the primary distribution. It is obvious that this method favors the small parties a lot. Dimitras (1991) points out that according to the 1989 system there was much less bias in favor of large parties. The procedure that has been used in the 1989 system for the secondary distribution is most proportional with respect to the procedure that the other systems use. This happens because large parties do not use all their votes in the secondary distribution and this means that they do not use the votes used in the primary allocation of the seats again. When all votes are used the quota is much larger than in the case that the remaining votes are used. However, a larger quota enables over-representation for large parties. Furthermore, small parties are more reinforced in the 1989 system because parties with at least one per cent of votes occupy at least one seat, and parties with at least 2% of votes take at least three seats. The secondary distribution, for 1974 up to 1895 systems, includes a special case for two-member districts. It was removed from the on1989 system but it was applied again in the 1993 electoral system. This is the following: If only one seat is allocated to a party in a primary distribution also the second seat is given to the same party only if half of its total votes is greater than the total votes for each one of the remaining parties. In fact this regulation is in a way the application of the PR d' Hont in the two-member districts: the largest value of $v_i/(s_i+1)$ is used , with s_i the number of seats received *so far* by the i^{th} party. For the party i^* that received the first seat the value of $v_{i*}/(s_{i*}+1) = v_{i*}/(1+1) = v_{i*}/2$, while for the rest parties $v_i/(0+1) = v_i$. The seat is given to the party i^*only if $v_{i*}/2 > v_i, \forall\ i$. Such restrictions

reveal the sparsity of the appearance of this case. As Pantellis and Triantafillou (1985) point out this case in fact appeared only one time, in 1963 in the lower district of Samos. In his opinion, this regulation was introduced after the interference of E.R.E, the political party that won the seat, and after that none has changed it. Thus, its omitting will not cause different results.

Tertiary distribution: All systems include a tertiary distribution of seats with the exception of 1989 system. Hare quota is applied throughout the state while the remaining seats are distributed according to the plurality rule. The 1985 system differs, as seats are allocated in the lower districts according to the relative majority in both lower districts and the entire state. Thus this rule is closer to the majority systems than to PR systems. Consequently, it favors the first party more and eliminates the possibility that small parties will gain seats in this distribution, because the number of seats which are allocated in this distribution are also eliminated.

State deputies' seats: In all cases Hare quota is used, as a first step, as total votes are divided by 12. In the first three systems the unlocated seats are distributed according to the largest remainders, while after 1985 d' Hont rule is applied.

As far as the thresholds are concerned, we have to point out the importance of the threshold of the 1993 system, which does not permit to small parties to gain a seat, in any district, in any distribution. These small parties are all parties with a total percentage of votes smaller than 3%. This means that these parties do not take a seat neither in the primary distribution nor in any of the following distributions. On the other hand, parties with a percentage of valid votes greater or equal to 3% of the total valid votes of all the parties, throughout the state, they obtain a minimum number of seats. This number is the integer part of the 70% of the seats that correspond to the percentage of a party's valid votes, multiplied by 300. These seats are eliminated from other parties according to the total number of seats. Thus, this regulation take seats from large parties and gives them to middle parties. The main rules of all systems are given in Table 4.

Table 4: The main rules, for all systems, for each one of the Primary (A), Secondary (B), Tertiary (C) and State Deputies (S. D.) distribution.

	A		**B**	**C**	**S.D.**
	$Magn. = 1$	$Magn. \neq 1$			
1974	*Rel. Maj.* ($d'Hond$)	*Hare*	*Hare* (17%)	*Hare* *remaining* : 1^{st} *party*	*Hare* *L.R*
1981	*Rel. Maj.* ($d'Hond$)	*Droop*	*Hare* (17%)	*Hare* *remaining* 1^{st} *party*	*Hare* *L.R.*
1985	*Rel.Maj.*	*Droop* *clause for* $'surpus'$ *seats*	*Hare*	*Rel.majority* ($lower + state$) *remaining* : 1^{st} *party*	*Hare* $d'Hont$
1989	*Rel. Maj.*	*Droop* *clause for* $'surpus'$ *seats*	*based on* *remainders* (1%, 2%)		*Hare* $d'Hont$
1993	*Rel. Maj.*	*Droop* *clause for* $'surpus'$ *seats*	*Hare* (3%, 70%)	*Hare*	*Hare* $d'Hont$

3.3 Electoral Formula's Algorithms

3.3.1 Notation

In this section each formula is performed with mathematical relations in the form of pseudo-algorithm. For this purpose we introduce the following notation:

l: lower electoral district, m: major electoral district, h: higher electoral district, s: state.

v: number of lower electoral districts.

i**:** counts the political parties.

p: number of political parties participating in the elections.

E_l: district magnitude of the lower district l.

E_{il}: district magnitude of the lower district l, for the party i.

E_{im}: district magnitude of the major district m, for the party i.

Ψ_{il}: total votes of party i in the lower district l.

Ψ_l: total votes in the lower district l. ($\Psi_l = \sum_{i=1}^{p} \Psi_{il}$).

$\sum_{l} \Psi_l$: total votes in the nation.

R_a: Remaining seats from the primary distribution determined in each major district.

R_{ab}: Remaining seats from the primary and the secondary distribution determined in the entire nation.

R_c: Remaining seats from the tertiary distribution determined in the entire nation.

R_s: Remaining seats from the distribution of the state deputes determined in the entire nation.

P_i: Percentage of votes for the party i, in the nation. ($P_i = \sum_{l=1}^{v} \Psi_{il} / \sum_{1=1}^{v} \Psi_l$).

P_i: Percentage of votes for the party i, in the lower district l. ($P_i = \Psi_{il} / \sum_{i=1}^{p} \Psi_{il}$).

M_a, M_b, M_c, M_s: Quotas for the primary, secondary, tertiary and the distribution of state deputies respectively.

Z_a, Z_b, Z_c, Z_s: The total number of seats given to parties in the primary, secondary, tertiary and the distribution of state deputies respectively.

Z_{ai}, Z_{bi}, Z_{ci}, Z_{si}: The total number of seats given to each party i in the primary, secondary, tertiary and the distribution of state deputies respectively.

n_i: describes the nature of the parties such that 1 represents autonomic parties, λ represents cartels of λ parties, (λ=2,3..) and 0 represents independent candidates.

I: the set of all parties participating in the primary, secondary, tertiary and the distribution of state deputies respectively.

T_i: total seats given to party i.

$\lfloor x \rfloor$: is the lower integer part of x, where x is a real number.

3.3.2 1974 algorithm

Primary Distribution of Seats (a)

1. calculate Hare quota: $M_a = \left\lfloor \frac{\Psi_l}{E_l} \right\rfloor$
2. computation of E_{il}:
   - if $E_l \neq 1$, then calculate district magnitude of the lower district l, for each party i: $E_{il} = \left\lfloor \frac{\Psi il}{Ma} \right\rfloor$
   - If $E_l = 1$, then we set $\Psi i^* l = \max_i \{\Psi il\}$ and $E_{il} = 0$ if $i \neq i^*$ and $E_{il} = 1$ if $i = i^*$
3. calculate the total number of seats given to all parties $Z_a = \sum_{l=1}^{v} \sum_{i=1}^{p} E_{il}$
4. calculate the total number of seats given to each party i, $Z_{ai} = \sum_{l=1}^{v} E_{il}$

Secondary Distribution of Seats (b)

1. First we define the set of parties i that consist I
   - if $n_i = 0 \Rightarrow i \notin I$
   - if $n_i = 1$ and $P_i \geq 17\% \Rightarrow i \in I$
   - if $n_i = 2$ and $P_i \geq 25\% \Rightarrow i \in I$
   - if $n_i > 2$ and $P_i \geq 30\% \Rightarrow i \in I$

2. If $|I| = 1$, then let i^* be the only political party such that $i^* \in I$. A second party i is included in I if and only if $\frac{P_i}{n_i} = \max_{i \neq i^*} \left\{ \frac{P_i}{n_i} \right\}$

3. If $|I| = 0$, then define δ_i such that:

   - $\delta_i = 17 - P_i$ if $n_i = 1$
   - $\delta_i = 25 - P_i$ if $n_i = 2$
   - $\delta_i = 30 - P_i$ if $n_i > 2$.
   - The set $\left\{ i', i^* \right\}$ is equal to I if and only if $\delta i^* = \max_i \{\delta i\}$ and $\delta i' = \max_{i \neq i^*} \{\delta i\}$

4. calculate quota $M_b = \left\lfloor \frac{\sum_{l=1}^{v} \sum_{i \in I} \Psi_{il}}{R_a} \right\rfloor$, where $R_a = \sum_{l \in M} E_l - \sum_{l \in M} \sum_{i=1}^{p} E_{il}$

5. calculate $E_{iM} = \left\lfloor \frac{\sum_{l \in M} \Psi il}{Mb} \right\rfloor$, $\forall\ i \in I$

6. calculate $Z_b = \sum_{M=1}^{9} \sum_{i \in I} E_{iM}$

7. calculate $Z_{bi} = \sum_{M=1}^{9} E_{iM}$ for each party i, $i \in I$.

Tertiary Primary Distribution of Seats (c)

1. evaluate $R_{ab} = 288 - Z_a - Z_b$

2. calculate quota $M_c = \left\lfloor \frac{\sum_{l=1}^{v} \sum_{i \in I} \Psi_{il}}{R_{ab}} \right\rfloor$ and $E_{hi} = \left\lfloor \frac{\sum_{l=1}^{v} \Psi il}{Mc} \right\rfloor$

3. calculate $Z_c = \sum_{i \in I} E_{hi}$ and $Z_{ci} = E_{hi}$

4. calculate $R_c = R_{ab} - \sum_{i \in I} E_{hi}$. If $R_c \neq 0$ we find the party i_0 such that $P_{i_0} = \max_i \{Pi\}$. We define $R_{ci} = R_c$ for $i = i_0$ and $R_{ci} = 0$ for $i \neq i_0$, thus each party takes $Z_{ci} + R_{ci}$ seats.

Distribution of the State Deputies Seats (s)

1. calculate quota $M_s = \left\lfloor \frac{\sum_{l=1}^{v} \sum_{i \in I} \Psi_{il}}{12} \right\rfloor$ and $E_{si} = \left\lfloor \frac{\sum_{l=1}^{v} E_{il}}{M_s} \right\rfloor$
2. calculate the remaining seats $R_s = 12 - \sum_{i \in I} E_{si}$.

   - If $R_s \neq 0$ the remaining seats are given according to the largest remainders.
   - For the party i the remainder is given by $\sum_{l=1}^{v} \Psi_{il} - E_{si} \cdot M_s$.
   - From the remainders each party i takes R_{si} seats. It is $Z_{si} = E_{si} + R_{si}$.

Finally, $T_i = Z_{ai} + Z_{bi} + Z_{ci} + R_{ci} + Z_{si}$.

3.3.3 1977 and 1981 algorithm

The only difference is in the quota of the primary distribution. Now droop quota is used instead of hare quota.

Droop quota: $M_a = \left\lfloor \frac{\Psi_l}{E_l + 1} \right\rfloor$.

3.3.4 1985 algorithm

Primary Distribution of Seats (a)

1. The distribution of seats is done according n_i :

   - if $n_i > 0$ then
     (a) calculate Hare quota: $M_{al} = \left\lfloor \frac{\Psi_l}{E_l} \right\rfloor$, for each lower district.
     (b) computation of E_{il}:
       - if $E_l \neq 1$, then calculate district magnitude of the lower district l, for each party i: $E_{il} = \left\lfloor \frac{\Psi il}{Ma} \right\rfloor$
       - If $E_l = 1$, then set $\Psi i^* l = \max_i \{\Psi il\}$ and $E_{il} = 0$ if $i \neq i^*$ and $E_{il} = 1$ if $i = i^*$

- if $n_i = 0$ then Ei$^* l = 1$ if and only if $\sum_{l=1}^{v} \Psi_{i^* l} > \sum_{l=1}^{v} \Psi$, $\forall\ i \neq i^*$.

2. Calculate the total number of seats given to all parties, in each district l: $Z_{al} = \sum_{i=1}^{p} E_{il}$

3. If there is l such that $Z_{al} \leq E_l$ then stop.

4. If there is l such that $Z_{al} > E_l$ then

   - Find i^* suct that $\{\Psi_{i^* l} - M_{al} * E_{i^* l}\} = \min \{\Psi_{il} - M_{al} * E_{il}\}, \forall i \neq i^*$ and then $E_{i^* l} = E_{i^* l} - 1$,
   - If (4) holds the procedure continious until $Z_{al} = E_l$, $\forall\ l$.

5. Calculate the total number of seats given to each party i, $Z_{ai} = \sum_{l=1}^{v} E_{il}$

6. Calculate the total number of seats given to each district l, $Z_{al} = \sum_{i=1}^{p} E_{il}$

7. Calculate the total number of seats given to all parties, $Z_a = \sum_{i=1}^{p} \sum_{l=1}^{v} E_{il}$

Secondary Distribution of Seats (b)

1. First we define the set of parties i that consist I

   - if $n_i = 0 \Rightarrow i \notin I$
   - if $n_i \neq 0 \Rightarrow i \in I$.

2. Calculate quota $M_b = \left\lfloor \frac{\sum_{l=1}^{v} \sum_{i \in I} \Psi il}{R_a} \right\rfloor$, for each Major district, where $R_a = \sum_{l \in M} E_l - \sum_{l \in M} \sum_{i=1}^{p} E_{il}$ the remaining indisposed seats from the primary distriution, in each Major district.

3. Calculate $E_{iM} = \left\lfloor \frac{\sum_{l \in M} \Psi il}{M_b} \right\rfloor$, $\forall\ i \in I$.

4. Calculate $Z_b = \sum_{M=1}^{9} \sum_{i \in I} E_{iM}$

5. Calculate $Z_{bi} = \sum_{M=1}^{9} E_{iM}$ for each party i, $i \in I$.

6. Allocation of seats of Major districts to Lower districts

   - compute the remaining indisposed seats from the primary distribution determined in each Lower district district, $R_{a_l} = E_l - Z_{al}$
   - compute the quota for each lower district l. $M_{b_l} = \left\lfloor \frac{\sum_{i \in I} \Psi_{il}}{R_{a_l}} \right\rfloor$
   - the number of seats for each party in each lower district is given by $E_{iM}^{elas} = \left\lfloor \frac{\Psi_{il}}{M_{b_l}} \right\rfloor$
   - the remaining seats from the above distribution are given by $R^{elas} = R_{a_l} - \sum_{i \in I} E_{il}^{elas}$
     (a) if $R^{elas} \neq 0$ then in each lower district is given R^{elas} seats. The first seat is given to the party i^* such that $\left\{ \Psi_{i^*l} - M_{b_l} * E_{i^*l}^{elas} \right\} = \max \left\{ \Psi_{il} - M_{b_l} * E_{il}^{elas} \right\}, \forall i \neq i^*$. The next seat is given to the party with the next maximum value of $\left\{ \Psi_{il} - M_{b_l} * E_{il}^{elas} \right\}$and so on. Let E_{il}^{upol} the seats given with this procedure.
     (b) if $R^{elas} = 0$ then noone party takes a seat.
   - If $E_l = 2$ and $\exists\ i^*$ such that $Z_{ai*} = 1$ then the only remaining seat is given to the party i^* if and only if $\frac{\sum_l \Psi_{i^*l}}{2} = \max \left\{ \sum_l \Psi_{il} \right\}, \forall i \neq i^*$
   - Total number of seats given up to now E_{iM}.
     (a) If $E_{iM} = E_{iM,}^{elas}\ \forall\ i, M$ then stop.
     (b) If $E_{iM} > E_{iM,}^{elas}\ \forall\ i, M$ then compute $\beta_{il} = \frac{\Psi il}{E_{il} + E_{il}^{upol}}$, for each party and for each lower district. For the party i that $\sum_l E_{il}^{upol} > \sum_M E_{iM}$ seats are substracting from the lower district l^* such that $\beta_{il^*} = \min \{\beta_{il^*}\}$ until $\sum_l E_{il}^{upol} > \sum_M E_{iM}$. Seats that were given at the 3rd and 5th substep of the 6th step of the algorithm are not substracting. The substracting seat is added in the same district $l^*$to the party i^*such that $\beta_{i^*l^*} = \max \{\beta_{i^*l^*}\}$ if and only if $\sum_{l \in M} E_{i^*l}^{upol} < \sum_M E_{i^*M}$.

Tertiary Primary Distribution of Seats (c)

1. Evaluate $R_{ab} = \sum_{l} E_l - Z_a - Z_b$
2. In each l : if there is party i_0 such that $\Psi i_0 l > \Psi il, \forall\, i \neq i_0$ and $\sum_{l} \Psi i_0 l > \sum_{l} \Psi il, \forall\, i \neq i_0$, this party takes all the remaining seats ϵ_l in l.
3. Calculate all the seats given with the above procedure $\epsilon_{\,i_o} = \sum_{l} \epsilon_{l\,i_o}$
4. Now the remaining seats are $R'_{ab} = R_{ab} - \epsilon_{\,i_o}$
5. Calculate quota $M_c = \left\lfloor \dfrac{\sum_{l=1}^{v} \sum_{i \in I} \Psi_{il}}{R'_{ab}} \right\rfloor$ and $E_{hi} = \left\lfloor \dfrac{\sum_{l=1}^{v} \Psi il}{M_c} \right\rfloor$
6. Calculate $Z_c = \sum_{i \in I} E_{hi} + \epsilon_{\,i_o}$ and $Z_{ci} = E_{hi} + \epsilon_{i_0}$
7. Calculate $R_c = R_{ab} - Z_c$ If $R_c \neq 0$ we find the party i_0 such that $P_{i_0} = \max_{i} \{Pi\}$. We define $R_{ci} = R_c$ for $i = i_0$ and $R_{ci} = 0$ for $i \neq i_0$, thus each party takes $Z_{ci} + R_{ci}$ seats.

Distribution of the State Deputies Seats (s)

1. Calculate quota $M_s = \left\lfloor \dfrac{\sum_{l=1}^{v} \sum_{i \in I} \Psi_{il}}{12} \right\rfloor$ and $E_{si} = \left\lfloor \dfrac{\sum_{l=1}^{v} E_{il}}{M_s} \right\rfloor$
2. Calculate the remaining seats $R_s = 12 - \sum_{i \in I} E_{si}$.
3. If $R_s \neq 0$ then compute $\lambda_i = \dfrac{\sum_{l=1}^{v} \Psi il}{1 + \sum_{i \in I} E_{si}}$
   - the first seat is given to the party i^*such that $\lambda_{i^*} = \max\{\lambda_i\}$, $\forall\, i \neq i^{*\cdot}$ The next seat is given to the party with the maximum λ_i and so on. From this procedure each party i takes R_{si} seats.
4. Computation of the total number of seat given to each party from the distribution (s) : $Z_{si} = E_{si} + R_{si}$.

Finally, $T_i = Z_{ai} + Z_{bi} + Z_{ci} + R_{ci} + Z_{si}$.

3.3.5 1989 algorithm

Primary Distribution of Seats (a)

1. The distribution of seats is done according n_i :

   - if $n_i > 0$ then

     (a) calculate Hare quota: $M_{al} = \left\lfloor \frac{\Psi_l}{E_l} \right\rfloor$, for each lower district.

     (b) computation of E_{il}:

     - if $E_l \neq 1$, then calculate district magnitude of the lower district l, for each party i: $E_{il} = \left\lfloor \frac{\Psi il}{Ma} \right\rfloor$
     - If $E_l = 1$, then set $\Psi i^* l = \max_i \{\Psi il\}$ and $E_{il} = 0$ if $i \neq i^*$ and $E_{il} = 1$ if $i = i^*$

   - if $n_i = 0$ then $\mathrm{Ei}^* l = 1$ if and only if $\sum_{l=1}^{v} \Psi_{i^* l} > \sum_{l=1}^{v} \Psi$, $\forall\ i \neq i^*$.

2. Calculate the total number of seats given to all parties, in each district l: $Z_{al} = \sum_{i=1}^{p} E_{il}$

3. If there is l such that $Z_{al} \leq E_l$ then stop.

4. If there is l such that $Z_{al} > E_l$ then

   - Find i^* suct that $\{\Psi_{i^* l} - M_{al} * E_{i^* l}\} = \min \{\Psi_{il} - M_{al} * E_{il}\}, \forall i \neq i^*$ and then $E_{i^* l} = E_{i^* l} - 1$,
   - If (4) holds the procedure continious until $Z_{al} = E_l$, $\forall\ l$.

5. Calculate the total number of seats given to each party i, $Z_{ai} = \sum_{l=1}^{v} E_{il}$

6. Calculate the total number of seats given to each party i,in each major district $Z_{aiM} = \sum_{l \in M} E_{il}$

7. Calculate the total number of seats given to each district l, $Z_{al} = \sum_{i=1}^{p} E_{il}$

8. Calculate the total number of seats given to all parties, $Z_a = \sum_{i=1}^{p} \sum_{l=1}^{v} E_{il}$

Secondary Distribution of Seats (b)

1. First we define the set of parties i that consist I

   - if $n_i = 0 \Rightarrow i \notin I$
   - if $n_i \neq 0 \Rightarrow i \in I$.

2. Calculate quota $M_b = \left\lfloor \dfrac{\sum\limits_{l=1}^{v} \sum\limits_{i \in I} \Psi^* il}{R_a} \right\rfloor$, for each Major district, where $R_a = \sum\limits_{l \in M} E_l - \sum\limits_{l \in M} \sum\limits_{i=1}^{p} E_{il}$ the remaining undisposed seats from the primary distriution, in each Major district, and $\Psi^* il$ are the remaining valid votes, the votes that have not been used for the seats alloacations in the primary distribution.

3. Calculate $E_{iM} = \left\lfloor \dfrac{\sum\limits_{l \in M} \Psi^* il}{M_b} \right\rfloor$, $\forall\ i \in I$.

4. Calculate $Z_b = \sum\limits_{M=1}^{9} \sum\limits_{i \in I} E_{iM}$

5. Calculate $Z_{bi} = \sum\limits_{M=1}^{9} E_{iM}$ for each party i, $i \in I$.

6. If there is M such that $E_{iM} < R_a$ then

   - Find i^* suct that $\{\Psi^*_{i^* l} - M_b * E_M\} = \max\{\Psi^*_{il} - M_b * E_M\}, \forall i \neq i^*$ and then $E_{i^* M} = E_{i^* M} + 1$,
   - If (6) holds the procedure continious until $E_{iM} = R_a$.

7. Let J be the set $\{i\prime\}$, where $i\prime$ satisfies the $\Psi_{i\prime} \succeq 0.02 \times \sum\limits_{l \in} \Psi il \ / \ \sum\limits_{l} \Psi_l$and $Z_{aiM} = 0$. If $i^* \in J$ then

   - If $i^* : \Psi i^* M = \max\{\Psi iM\}, i \in J$, then $E_{i^* M} = E_{iM} + 1$.
   - This procedure continious untill $E_{iM} = 3$, for each $i \in J$.

8. Let H be the set $\{i\prime\}$, where $i\prime$ satisfies the $\Psi_{i\prime} \succeq 0.01 \times \sum\limits_{l \in} \Psi il \ / \ \sum\limits_{l} \Psi_l$ and $\Psi_{i\prime} \preceq 0.02 \times \sum\limits_{l \in} \Psi il \ / \ \sum\limits_{l} \Psi_l$. If $i^* \in H$ then

- If i^*, in the major district M^* satisfies the $\Psi i^* M^* = \max\{\Psi iM\}, i \in J$, then $E_{i^*M^*} = 1$.

9. Let R_M be the available seats in each major district. If $R_M \neq 0$ then

   - Calculate $M'_b = \left\lfloor \frac{\sum_{l=1}^{v} \sum_{i \in I} \Psi^* il}{R_M} \right\rfloor$ and $E'_{iM} = \left\lfloor \frac{\sum_{l \in M} \Psi^* il}{M'_b} \right\rfloor$

10. If there are still available seats R'_M then find i^* suct that $\left\{ \sum_{l \in M} \Psi^* i^* l - M'_b * E'_{i^*M} \right\} = \max \left\{ \sum_{l \in M} \Psi^* il - M'_b * E_{iM} \right\}, \forall i \neq i^*$ and then $E'_{i^*M} = E'_{i^*M} + 1$, until all seats are distributed.

11. The total seats given to i, from (b), in the nation, are $Z_{bi} = \sum_{M} E_{iM} + E'_{iM}$

Distribution of the State Deputies Seats (s)

1. Calculate quota $M_s = \left\lfloor \frac{\sum_{l=1}^{v} \sum_{i \in I} \Psi_{il}}{12} \right\rfloor$ and $E_{si} = \left\lfloor \frac{\sum_{l=1}^{v} E_{il}}{M_s} \right\rfloor$

2. Calculate the remaining seats $R_s = 12 - \sum_{i \in I} E_{si}$.

3. If $R_s \neq 0$ then compute $\lambda_i = \frac{\sum_{l=1}^{v} \Psi il}{1 + \sum_{i \in I} E_{si}}$

   - the first seat is given to the party i^*such that $\lambda_{i^*} = \max\{\lambda_i\}$, $\forall\ i \neq i^{*\cdot}$ The next seat is given to the party with the maximum λ_i and so on. From this procedure each party i takes R_{si} seats.

4. Computation of the total number of seat given to each party from the distribution (s) : $Z_{si} = E_{si} + R_{si}$.

Finally, $T_i = Z_{ai} + Z_{bi} + Z_{si}$.

3.3.6 1993 and 1996 algorithm

Primary Distribution of Seats (a)

1. The distribution of seats is done according n_i :

   - if $n_i > 0$ then

     (a) calculate Hare quota: $M_{al} = \left\lfloor \frac{\Psi_l}{E_l} \right\rfloor$, for each lower district.

     (b) computation of E_{il}:

     - if $E_l \neq 1$, then calculate district magnitude of the lower district l, for each party i: $E_{il} = \left\lfloor \frac{\Psi il}{Ma} \right\rfloor$
     - If $E_l = 1$, then set $\Psi i^* l = \max_i \{\Psi il\}$ and $E_{il} = 0$ if $i \neq i^*$ and $E_{il} = 1$ if $i = i^*$

   - if $n_i = 0$ then Ei* $l = 1$ if and only if $\sum_{l=1}^{v} \Psi_{i^* l} > \sum_{l=1}^{v} \Psi$, $\forall\ i \neq i^*$.

2. Calculate the total number of seats given to all parties, in each district l: $Z_{al} = \sum_{i=1}^{p} E_{il}$

3. If there is l such that $Z_{al} \leq E_l$ then stop.

4. If there is l such that $Z_{al} > E_l$ then

   - Find i^* suct that $\{\Psi_{i^* l} - M_{al} * E_{i^* l}\} = \min \{\Psi_{il} - M_{al} * E_{il}\}, \forall i \neq i^*$ and then $E_{i^* l} = E_{i^* l} - 1$,
   - If (4) holds the procedure continious until $Z_{al} = E_l$, $\forall\ l$.

5. Calculate the total number of seats given to each party i, $Z_{ai} = \sum_{l=1}^{v} E_{il}$

6. Calculate the total number of seats given to each district l, $Z_{al} = \sum_{i=1}^{p} E_{il}$

7. Calculate the total number of seats given to all parties, $Z_a = \sum_{i=1}^{p} \sum_{l=1}^{v} E_{il}$

Secondary Distribution of Seats (b)

1. First we define the set of parties i that consist I

   - if $n_i = 0 \Rightarrow i \notin I$
   - if $n_i \neq 0 \Rightarrow i \in I$.

2. Calculate quota $M_b = \left\lfloor \frac{\sum_{l=1}^{v} \sum_{i \in I} \Psi_{il}}{R_a} \right\rfloor$, where $R_a = \sum_{l \in M} E_l - \sum_{l \in M} \sum_{i=1}^{p} E_{il}$

3. calculate $E_{iM} = \left\lfloor \frac{\sum_{l \in M} \Psi il}{Mb} \right\rfloor$, $\forall\ i \in I$

4. calculate $Z_b = \sum_{M=1}^{9} \sum_{i \in I} E_{iM}$

5. calculate $Z_{bi} = \sum_{M=1}^{9} E_{iM}$ for each party i, $i \in I$.

Allocation of seats of Major districts to Lower districts

- compute the remaining indisposed seats from the primary distribution determined in each Lower district district, $R_{a_l} = E_l - Z_{al}$
- compute the quota for each lower district l. $M_{b_l} = \left\lfloor \frac{\sum_{i \in I} \Psi_{il}}{R_{a_l}} \right\rfloor$
- the number of seats for each party in each lower district is given by $E_{iM}^{elas} = \left\lfloor \frac{\Psi_{il}}{M_{b_l}} \right\rfloor$
- the remaining seats from the above distribution are given by $R^{elas} = R_{a_l} - \sum_{i \in I} E_{il}^{elas}$
  (a) if $R^{elas} \neq 0$ then in each lower district is given R^{elas} seats. The first seat is given to the party i^* such that $\left\{\Psi_{i^*l} - M_{b_l} * E_{i^*l}^{elas}\right\} = \max\left\{\Psi_{il} - M_{b_l} * E_{il}^{elas}\right\}, \forall i \neq i^*$. The next seat is given to the party with the next maximum value of $\left\{\Psi_{il} - M_{b_l} * E_{il}^{elas}\right\}$and so on. Let E_{il}^{upol} the seats given with this procedure.
  (b) if $R^{elas} = 0$ then noone party takes a seat.
- If $E_l = 2$ and $\exists\ i^*$ such that $Z_{ai*} = 1$ then the only remaining seat is given to the party i^* if and only if $\frac{\sum_l \Psi_{i^*l}}{2} = \max\left\{\sum_l \Psi_{il}\right\}, \forall i \neq i^*$

- Total number of seats given up to now E_{iM}.
  (a) If $E_{iM} = E^{elas}_{iM,}\ \forall\ i, M$ then stop.
  (b) If $E_{iM} > E^{elas}_{iM,}\ \forall\ i, M$ then compute $\beta_{il} = \frac{\Psi il}{E_{il}+E^{upol}_{il}}$, for each party and for each lower district. For the party i that $\sum\limits_{l} E^{upol}_{il} > \sum\limits_{M} E_{iM}$ seats are substracting from the lower district l^* such that $\beta_{il^*} = \min\{\beta_{il^*}\}$ until $\sum\limits_{l} E^{upol}_{il} > \sum\limits_{M} E_{iM}$. Seats that were given at the 3rd and 5th substep of the 6th step of the algorithm are not substracting. The substracting seat is added in the same district $l^*$to the party i^*such that $\beta_{i^*l^*} = \max\{\beta_{i^*l^*}\}$ if and only if $\sum\limits_{l\in M} E^{upol}_{i^*l} < \sum\limits_{M} E_{i^*M}$.

Tertiary Primary Distribution of Seats (c)

1. evaluate $R_{ab} = 288 - Z_a - Z_b$
2. calculate quota $M_c = \left\lfloor \frac{\sum\limits_{l=1}^{v}\sum\limits_{i\in I}\Psi_{il}}{R_{ab}} \right\rfloor$ and $E_{hi} = \left\lfloor \frac{\sum\limits_{l=1}^{v}\Psi il}{Mc} \right\rfloor$
3. calculate $Z_c = \sum\limits_{i\in I} E_{hi}$ and $Z_{ci} = E_{hi}$
4. calculate $R_c = R_{ab} - \sum\limits_{i\in I} E_{hi}$. If $R_c \neq 0$ we find the party i_0 such that $P_{i_0} = \max\limits_{i}\{Pi\}$. We define $R_{ci} = R_c$ for $i = i_0$ and $R_{ci} = 0$ for $i \neq i_0$, thus each party takes $Z_{ci} + R_{ci}$ seats.

Distribution of the State Deputies Seats (s)

1. Calculate quota $M_s = \left\lfloor \frac{\sum\limits_{l=1}^{v}\sum\limits_{i\in I}\Psi_{il}}{12} \right\rfloor$ and $E_{si} = \left\lfloor \frac{\sum\limits_{l=1}^{v} E_{il}}{M_s} \right\rfloor$
2. Calculate the remaining seats $R_s = 12 - \sum\limits_{i\in I} E_{si}$.
3. If $R_s \neq 0$ then compute $\lambda_i = \frac{\sum\limits_{l=1}^{v}\Psi il}{1+\sum\limits_{i\in I} E_{si}}$
   - the first seat is given to the party i^*such that $\lambda_{i^*} = \max\limits_{i}\{\lambda_i\}$, $\forall\ i \neq i^{*\cdot}$ The next seat is given to the party with the maximum λ_i and so on. From this procedure each party i takes R_{si} seats.

4. Computation of the total number of seat given to each party from the distribution (s) : $Z_{si} = E_{si} + R_{si}$.

5. Compute $T_i = Z_{ai} + Z_{bi} + Z_{si}$, for each party i.

6. Let K be the set $\{i\}$, where i satisfies the $\Psi_{i\prime} \preceq 0.03 \sum_{l\in} \Psi il$. If $i^* \in K$ then $T_{i^*} = 0$.

7. Let L be the set $\{i\}$, where i satisfies the $\Psi_{i\prime} \geq 0.03 \sum_{l\in} \Psi il$. If $i^* \in L$ then T_{i^*} must be $T_{i^*} \geq 210 \times \Psi_i \ / \sum_l \Psi_l = Ul_{i^*}$. For this purpose

   - If $i \in L$ and $T_i \leq Ul_i$ then set $T_i = Ul_{i.}$
   - The $\sum_i Ul_i$ seats are taken from other parties. The first is subtracted from the party i' with $\Psi_{i'} = \underset{i}{max}\mathrm{I}\{\Psi_i\}$, the second from the party i'' with $\Psi_{i''} = \underset{i\ -\ i'}{max}\{\Psi_i\}$, and so on.