# Package 'bivpois'

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Title Bivariate Poisson Models Using The EM Algorithm

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**Depends** R (>= 2.0.1)

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**Description** Functions for fitting Bivariate Poisson Models using the EM algorithm. Details can be found in Karlis and Ntzoufras (2003, RSS D & 2004, AUEB Technical Report)

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URL http://www.stat-athens.aueb.gr/~jbn/papers/paper14.htm

## R topics documented:

bivpois.table Probability of Bivariate Poisson Using Recursive relations

#### Description

Returns the probability of the bivariate Poisson distribution using recursive relations.

## Usage

bivpois.table(x, y, lambda = c(1, 1, 1))

#### Arguments

х, у	single values containing which values should evaluated (x and y should be at least 1)
lambda	Vector (of length 3) containing values of the parameters lambda1, lambda2 and lambda3 of the bivariate poisson distribution.

#### Details

In order to calculate bivpoison probability values we use recursive relationships. This function is much slower than pbivpois

## Value

A matrix with dimension (x+1) X (y+1) is returned. Cell ij contains the probability P(X=i-1, Y=j-1).

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#### References

1. Karlis, D. and Ntzoufras, I. (2005). Bivariate Poisson and Diagonal Inflated Bivariate Poisson Regression Models in R. Journal of Statistical Software (to appear).

2. Karlis, D. and Ntzoufras, I. (2003). Analysis of Sports Data Using Bivariate Poisson Models. Journal of the Royal Statistical Society, D, (Statistician), 52, 381 - 393.

## See Also

pbivpois. simple.bp. lm.bp. lm.dibp.

ex1.sim Bivpois Example 1 Dataset: Simulated Data	
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## Description

The data has one pair (x, y) of bivariate Poisson variables and five variables  $(z_1, \ldots, z_5)$  generated from N(0, 0.01) distribution. Hence

 $X_{i}, Y_{i} \sim BP(\lambda_{1i}, \lambda_{2i}, \lambda_{3i}) \text{ with}$  $\log \lambda_{1i} = 1.8 + 2Z_{1i} + 3Z_{3i}$  $\log \lambda_{2i} = 0.7 - Z_{1i} - 3Z_{3i} + 3Z_{5i}$  $\log \lambda_{3i} = 1.7 + Z_{1i} - 2Z_{2i} + 2Z_{3i} - 2Z_{4i}.$ 

#### Usage

data(ex1.sim)

#### Format

A data frame with 100 observations on the following 7 variables.

x,y Simulated Bivariate Poisson Variables used as response

z1,z2,z3,z4,z5 Simulated N(0,0.01) explanatory variables

ex1.sim

#### Details

This data is used as example one in Karlis and Ntzoufras (2004).

#### Source

1. Karlis, D. and Ntzoufras, I. (2005). Bivariate Poisson and Diagonal Inflated Bivariate Poisson Regression Models in R. Journal of Statistical Software (to appear).

#### References

Karlis, D. and Ntzoufras, I. (2003). Analysis of Sports Data Using Bivariate Poisson Models. Journal of the Royal Statistical Society, D, (Statistician), 52, 381 - 393.

#### Examples

```
#
# Double and Bivariate Poisson models can be fitted using the command
# demo(ex1, package='bivpois')
#
# Here we present the same commands but iterations of the EM were restricted to 2 to save time
             s) # load bivpois library
# load data of example 1
library(bivpois)
data(ex1.sim)
# ------
# Simple Bivariate Poisson Model
ex1.simple<-simple.bp( ex1.sim$x, ex1.sim$y, maxit=2) # fit simple model of section 4.1.1
                   # monitor output variables
names(ex1.simple)
ex1.simple$lambda
                   # view lambda1
ex1.simple$BIC
                   # view BIC
ex1.simple
                    # view all results of the model
# Fit Double and Bivariate Poisson models ()
# Model 2: DblPoisson(11, 12)
ex1.m2<-lm.bp(x~1 , y~1 , data=ex1.sim, zeroL3=TRUE)</pre>
# Model 3: BivPoisson(11, 12, 13); same as simple.bp(ex1.sim$x, ex1.sim$y)
ex1.m3<-lm.bp(x~1 , y~1 , data=ex1.sim, maxit=2)</pre>
# Model 4: DblPoisson (l1=Full, l2=Full)
ex1.m4<-lm.bp(x~., y~., data=ex1.sim, zeroL3=TRUE)</pre>
# Model 5: BivPoisson(11=full, 12=full, 13=constant)
ex1.m5<-lm.bp(x~., y~., data=ex1.sim, maxit=2)</pre>
# Model 6: DblPois(11,12)
ex1.m6<-lm.bp(x~z1 , y~z1+z5 , l112=~z3, data=ex1.sim, zeroL3=TRUE)
# Model 7: BivPois(11,12,13=constant)
ex1.m7<-lm.bp(x~z1 , y~z1+z5 , l1l2=~z3, data=ex1.sim, maxit=2)
# Model 8: BivPoisson(11=full, 12=full, 13=full)
ex1.m8<-lm.bp(x~., y~., 13=~., data=ex1.sim, maxit=2)
# Model 9: BivPoisson(11=full, 12=full, 13=z1+z2+z3+z4)
ex1.m9<-lm.bp(x~., y~., 13=~.-z5, data=ex1.sim, maxit=2)
# Model 10: BivPoisson(11, 12, 13=full)
ex1.m10<-lm.bp(x~z1 , y~z1+z5 , l112=~z3, l3=~., data=ex1.sim, maxit=2)
# Model 11: BivPoisson(11, 12, 13= z1+z2+z3+z4)
ex1.m11<-lm.bp(x~z1 , y~z1+z5 , l1l2=~z3, l3=~.-z5, data=ex1.sim, maxit=2)
#
```

ex2.sim

```
ex1.m11$coef # monitor all beta parameters of model 11
#
ex1.m11$beta1 # monitor all beta parameters of lambda1 of model 11
ex1.m11$beta2 # monitor all beta parameters of lambda2 of model 11
ex1.m11$beta3 # monitor all beta parameters of lambda3 of model 11
```

ex2.sim

Bivpois Example 2 Dataset: Simulated Data

## Description

The data has one pair (x, y) of diagonal inflated bivariate Poisson variables and five variables  $(z_1, \ldots, z_5)$  generated from N(0, 0.12) distribution. Hence

 $\begin{aligned} X_i, Y_i &\sim DIBP(\lambda_{1i}, \lambda_{2i}, \lambda_{3i}, p = 0.30, Poisson(2)) \text{ with} \\ &\log \lambda_{1i} = 1.8 + 2Z_{1i} + 3Z_{3i} \\ &\log \lambda_{2i} = 0.7 - Z_{1i} - 3Z_{3i} + 3Z_{5i} \\ &\log \lambda_{3i} = 1.7 + Z_{1i} - 2Z_{2i} + 2Z_{3i} - 2Z_{4i}. \end{aligned}$ 

Usage

data(ex2.sim)

#### Format

A data frame with 100 observations on the following 7 variables.

x,y Simulated Bivariate Poisson Variables used as response

z1,z2,z3,z4,z5 Simulated N(0,0.01) explanatory variables

## Details

This data is used as example one in Karlis and Ntzoufras (2004).

## Source

1. Karlis, D. and Ntzoufras, I. (2005). Bivariate Poisson and Diagonal Inflated Bivariate Poisson Regression Models in R. Journal of Statistical Software (to appear).

#### References

Karlis, D. and Ntzoufras, I. (2003). Analysis of Sports Data Using Bivariate Poisson Models. Journal of the Royal Statistical Society, D, (Statistician), 52, 381 - 393.

#### ex3.health

#### Examples

```
#
   Models of example 2 can be fitted using the command
#
   demo(ex2, package='bivpois')
#
#
  Here we present the same commands but iterations of the EM were restricted to 2 to save time
library(bivpois) # load bivpois library
data(ex2.sim)
                  # load ex2.sim data from bivpois library
# Model 1: BivPois
ex2.m1<-lm.bp( x~z1 , y~z1+z5, l1l2=~z3, l3=~.-z5, data=ex2.sim, maxit=2 )
# Model 2: Zero Inflated BivPois
ex2.m2<-lm.dibp( x~z1 , y~z1+z5, l1l2=~z3, l3=~.-z5, data=ex2.sim , jmax=0, maxit=2 )
# Model 3: Diagonal Inflated BivPois with DISCRETE(1) diagonal distribution
ex2.m3<-lm.dibp( x<sup>2</sup>1 , y<sup>2</sup>1+z5, 1112=<sup>2</sup>z3, 13=<sup>2</sup>.-z5, data=ex2.sim , jmax=1, maxit=2 )
# Model 4: Diagonal Inflated BivPois with DISCRETE(2) diagonal distribution
ex2.m4<-lm.dibp( x<sup>2</sup>1 , y<sup>2</sup>1+z5, 1112=<sup>2</sup>z3, 13=<sup>2</sup>.-z5, data=ex2.sim , jmax=2, maxit=2 )
# Model 5: Diagonal Inflated BivPois with DISCRETE(3) diagonal distribution
ex2.m5<-lm.dibp( x<sup>2</sup>1 , y<sup>2</sup>1+z5, 1112=<sup>2</sup>z3, 13=<sup>2</sup>.-z5, data=ex2.sim , jmax=3, maxit=2 )
# Model 6: Diagonal Inflated BivPois with DISCRETE(4) diagonal distribution
ex2.m6<-lm.dibp( x<sup>2</sup>1 , y<sup>2</sup>1+z5, 1112=<sup>2</sup>z3, 13=<sup>2</sup>.-z5, data=ex2.sim , jmax=4, maxit=2 )
# Model 7: Diagonal Inflated BivPois with DISCRETE(5) diagonal distribution
ex2.m7<-lm.dibp( x<sup>2</sup>1 , y<sup>2</sup>1+z5, 1112=<sup>2</sup>z3, 13=<sup>2</sup>.-z5, data=ex2.sim , jmax=5, maxit=2 )
# Model 8: Diagonal Inflated BivPois with DISCRETE(6) diagonal distribution
ex2.m8<-lm.dibp( x~z1 , y~z1+z5, l112=~z3, l3=~.-z5, data=ex2.sim , jmax=6, maxit=2 )
# Model 9: Diagonal Inflated BivPois with POISSON diagonal distribution
ex2.m9<-lm.dibp( x~z1 , y~z1+z5, l112=~z3, l3=~.-z5, data=ex2.sim ,
                   distribution="poisson", maxit=2 )
# Model 10: Diagonal Inflated BivPois with GEOMETRIC diagonal distribution
ex2.m10<-lm.dibp( x~z1 , y~z1+z5, l112=~z3, l3=~.-z5, data=ex2.sim ,
                    distribution="geometric", maxit=2 )
#
# printing parameters of model 7
ex2.m7$beta1
ex2.m7$beta2
ex2.m7$beta3
ex2.m7$p
ex2.m7$theta
# printing parameters of model 9
ex2.m9$beta1
ex2.m9$beta2
ex2.m9$beta3
ex2.m9$p
ex2.m9$theta
```

```
ex3.health
```

Bivpois Example 3 Dataset: Health Care Data

## Description

Demand for health care in Australia data (Cameron and Trivedi, 1986). The data refer to the Australian Health survey for 1977-1978 with sample size equal to 5190.

#### Usage

data(ex3.health)

#### Format

A data frame with 5190 observations on the following 20 variables.

doctorco Number of consultations with a doctor or specialist in the past 2 weeks

prescrib Total number of prescribed medications used in past 2 days

sex 1 if female, 0 if male

- **age** Age in years divided by 100 (measured as mid-point of 10 age groups from 15-19 years to 65-69 with 70 or more coded treated as 72)
- agesq AGE squared
- income Annual income in Australian dollars divided by 1000 (measured as mid-point of coded ranges Nil, <200, 200-1000, 1001-, 2001-, 3001-, 4001-, 5001-, 6001-, 7001-, 8001-10000, 10001-12000, 12001-14000, with 14001- treated as 15000 ).</p>

**levyplus** 1 if covered by private health insurance fund for private patient in public hospital (with doctor of choice), 0 otherwise

- **freepoor** 1 if covered by government because low income, recent immigrant, unemployed, 0 otherwise
- freepera 1 if covered by government because low income, recent immigrant, unemployed, 0 otherwise
- illness Number of illnesses in past 2 weeks with 5 or more coded as 5
- actdays Number of days of reduced activity in past two weeks due to illness or injury
- **hscore** General health questionnaire score using Goldberg's method. High score indicates bad health.
- chcond1 1 if chronic condition(s) but not limited in activity, 0 otherwise
- chcond2 1 if chronic condition(s) and limited in activity, 0 otherwise
- **nondocco** Number of consultations with non-doctor health professionals (chemist, optician, physiotherapist, social worker, district community nurse, chiropodist or chiropractor) in the past 2 weeks
- hospadmi Number of admissions to a hospital, psychiatric hospital, nursing or convalescent home in the past 12 months (up to 5 or more admissions which is coded as 5)
- hospdays Number of nights in a hospital, etc. during most recent admission: taken, where appropriate, as the mid-point of the intervals 1, 2, 3, 4, 5, 6, 7, 8-14, 15-30, 31-60, 61-79 with 80 or more admissions coded as 80. If no admission in past 12 months then equals zero.
- medicine Total number of prescribed and nonprescribed medications used in past 2 days

nonpresc Total number of nonprescribed medications used in past 2 days

constant Constant term

#### Details

Details can be found in Cameron and Trivedi (1986). This data is used as example three in Karlis and Ntzoufras (2005). In this illustration two variables are used as response: the number of consultations with a doctor or a specialist and the total number of prescribed medications used in past 2 days (doctorco, prescrib). Three variables have been used as covariates: the gender (1 female, 0 male), the age in years divided by 100 (measured as midpoints of age groups) and the annual income in Australian dollars divided by 1000 (measured as midpoint of coded ranges) sex, age, income.

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#### Source

Cameron, A.C. and Trivedi, P.K. (1986). Econometric Models Based on Count Data: Comparisons and Applications of Some Estimators and Tests, Journal of Applied Econometrics, 1, 29 - 54.

#### References

1. Cameron, A.C., Trivedi, P.K., Milne, F. and Piggott, J. (1988). A Microeconometric Model of the Demand for Health Care and Health Insurance in Australia, Review of Economic Studies, 55, 85 - 106.

2. Cameron, A.C. and Trivedi, P.K. (1993). Tests of Independence in Parametric Models with Applications and Illustrations, Journal of Business & Economics Statistics, 11, 29 - 43.

3. Karlis, D. and Ntzoufras, I. (2005). Bivariate Poisson and Diagonal Inflated Bivariate Poisson Regression Models in R. Journal of Statistical Software (to appear).

4. Karlis, D. and Ntzoufras, I. (2003). Analysis of Sports Data Using Bivariate Poisson Models. Journal of the Royal Statistical Society, D, (Statistician), 52, 381 - 393.

#### Examples

```
#
   Models of example 3 can be fitted using the command
  demo(ex3, package='bivpois')
#
#
#
  Here we present the commands for the same models commented out in order to save time
#
#library(bivpois)
#data(ex3.health)
# Bivariate Poisson models
#ex3.model.a<-lm.bp(doctorco~sex+age+income, prescrib~sex+age+income,</pre>
                     data=ex3.health)
#ex3.model.b<-lm.bp(doctorco~sex+age+income, prescrib~sex+age+income, 13=~sex,</pre>
                     data=ex3.health)
# Double Poisson model
#ex3.model.c<-lm.bp(doctorco sex+age+income, prescrib sex+age+income,</pre>
                     data=ex3.health, zeroL3=TRUE)
# diagonal inflated models
#ex3.dibp.a<-lm.dibp(doctorco sex+age+income, prescrib sex+age+income)</pre>
                      data=ex3.health, jmax=0)
                                                                          # model (a)
#ex3.dibp.b<-lm.dibp(doctorco~sex+age+income, prescrib~sex+age+income,13=~sex,</pre>
#
                      data=ex3.health, jmax=0)
                                                                          # model (b)
```

ex4.ita91

Bivpois Example 4 Dataset: Italian Series A Football Scores for Season 1991-92

#### Description

Italian Serie A football scores for season 1991-92.

#### Usage

data(ex4.ita91)

#### Format

A data frame with 306 observations on the following 4 variables.

- g1 Goals scored by the home team
- $\mathbf{g2}$  Goals scored by the away team
- team1 a factor indicating the home team with levels Ascoli Atalanta Bari Cagliari Cremonese Fiorentina Foggia Genoa Inter Juventus Lazio Milan Napoli Parma Roma Sampdoria Torino Verona
- team2 a factor indicating the away team with levels Ascoli Atalanta Bari Cagliari Cremonese Fiorentina Foggia Genoa Inter Juventus Lazio Milan Napoli Parma Roma Sampdoria Torino Verona

#### Details

Data were originally used in Karlis and Ntzoufras (2003). The data consist of pairs of counts indicating the number of goals scored by each of the two competing teams. As covariates we have used dummy variables to model the team strength. In modelling outcomes of football games, it has been observed an excess of draws and small over-dispersion. Introducing diagonal inflated models we correct for both the over-dispersion and the excess of draws.

#### Source

Karlis, D. and Ntzoufras, I. (2003). Analysis of Sports Data Using Bivariate Poisson Models. Journal of the Royal Statistical Society, D, (Statistician), 52, 381 - 393.

#### References

1. Karlis, D. and Ntzoufras, I. (2005). Bivariate Poisson and Diagonal Inflated Bivariate Poisson Regression Models in R. Journal of Statistical Software (to appear).

## Examples

```
# Models 1-12 of example 4 can be fully reproduced using the command
#
  demo(ex4, package='bivpois')
#
#
  Here we present the same commands but iterations of the EM were restricted to 10 to save time
#
# Models 1-12 can be run using the demo command demo(ex4,package='bivpois')
#
library(bivpois) # loading of bivpois library
data(ex4.ita91) # loading ex4.ita91 data from bivpois library
# formula for modeling of lambda1 and lambda2
form1 <- ~c(team1,team2)+c(team2,team1)</pre>
# Model 1: Double Poisson
ex4.m1<-lm.bp( g1~1, g2~1, l1l2=form1, zeroL3=TRUE, data=ex4.ita91)
# Models 2-5: bivariate Poisson models
ex4.m2<-lm.bp(g1~1,g2~1, l1l2=form1, data=ex4.ita91, maxit=2)
ex4.m3<-lm.bp(g1~1,g2~1, l1l2=form1, l3=~team1, data=ex4.ita91, maxit=2)
ex4.m4<-lm.bp(g1~1,g2~1, l1l2=form1, l3=~team2, data=ex4.ita91, maxit=2)
```

```
ex4.m5<-lm.bp(g1~1,g2~1, l1l2=form1, l3=~team1+team2, data=ex4.ita91, maxit=2)
# Model 6: Zero Inflated Model
ex4.m6 <-lm.dibp(g1~1,g2~1, l112=form1, data=ex4.ita91, jmax=0, maxit=2)
# Models 7-11: Diagonal Inflated Bivariate Poisson Models
ex4.m7 <-lm.dibp(g1~1,g2~1, l1l2=form1, data=ex4.ita91, distribution="geometric" , maxit=2)</pre>
ex4.m8 <-lm.dibp(g1~1,g2~1, l1l2=form1, data=ex4.ita91, jmax=1, maxit=2)
ex4.m9 <-lm.dibp(g1~1,g2~1, l1l2=form1, data=ex4.ita91, jmax=2, maxit=2)
ex4.m10<-lm.dibp(g1~1,g2~1, l1l2=form1, data=ex4.ita91, jmax=3, maxit=2)
ex4.m11<-lm.dibp(g1~1,g2~1, l1l2=form1, data=ex4.ita91, distribution="poisson", maxit=2)
# Models 12: Diagonal Inflated Double Poisson Model
ex4.m12 <- lm.dibp( g1~1,g2~1, l1l2=form1, data=ex4.ita91, distribution="poisson",
                 zeroL3=TRUE , maxit=2)
# -----
#
   # --
# monitoring parameters for model 1: Dbl Poisson
ex4.m1$coef
             # all parameters
ex4.m1$beta1
             # model parameters for lambda1
ex4.m1$beta2  # model parameters for lambda2.
             # All are the same as in beta1 except the intercept
ex4.m1$beta2[1] # Intercpept for lambda2.
ex4.m1$beta2[1]-ex4.m1$beta2[2] # estimated home effect
# estimating the effect for 18th level of attack (team1..team2) [Verona]
-sum(ex4.m1$coef[ 2:18])
# estimating the effect for 18th level of defence(team2..team1) [Verona]
-sum(ex4.m1$coef[19:35])
# monitoring parameters for model 2: BivPoisson(lamdba1,lambda2,constant lamdba3)
#
# monitoring parameters for model 1: Dbl Poisson
ex4.m2$beta1  # model parameters for lambda1
ex4.m2$beta2
              # model parameters for lambda2.
              # All are the same as in beta1 except the intercept
ex4.m2$beta3
ex4.m2$beta3  # model parameters for lambda3 (Here only the intercept)
ex4.m2$beta2[1]  # Intercept for lambda2.
ex4.m2$beta2[1]-ex4.m2$beta2[2] # estimated home effect
# estimating the effect for 18th level of attack (team1..team2) [Verona]
-sum(ex4.m2$coef[ 2:18])
# estimating the effect for 18th level of defence(team2..team1) [Verona]
-sum(ex4.m2$coef[19:35])
#
                 #
# monitoring parameters for model 8: Biv.Poisson with Dis(1) diagonal distribution
#
#
# monitoring parameters for model 1: Dbl Poisson
ex4.m8$beta1  # model parameters for lambda1
```

```
ex4.m8$beta2
                  # model parameters for lambda2.
                  # All are the same as in beta1 except the intercept
ex4.m8$beta3
                  # model parameters for lambda3. Here beta3 has only the intercept
ex4.m8$beta2[1] # Intercpept for lambda2.
ex4.m8$beta2[1]-ex4.m8$beta2[2] # estimated home effect
# estimating the effect for 18th level of attack (team1..team2) [Verona]
-sum(ex4.m8$coef[ 2:18])
# estimating the effect for 18th level of defence(team2..team1) [Verona]
-sum(ex4.m8$coef[19:35])
ex4.m8$beta3
                             # parameters for lambda3 (here the intercept)
exp(ex4.m8$beta3)
                             # lambda3 (here constant)
ex4.m8$diagonal.distribution # printing details for the diagonal distribution
ex4.m8$p
                             # mixing proportion
ex4.m8$theta
                             # printing theta parameters
```

lm.bp

General Bivariate Poisson Model

#### Description

Produces a "list" object which gives details regarding the fit of a bivariate Poisson regression model of the form

$$X_i, Y_i \sim BP(\lambda_{1i}, \lambda_{2i}, \lambda_{3i})$$
 for  $i = 1, 2, \dots, n$ , with

$$\log \underline{\lambda}_1 = \mathbf{w}_1 \underline{\beta}_1, \ \log \underline{\lambda}_2 = \mathbf{w}_2 \underline{\beta}_2 \text{ and } \log \underline{\lambda}_3 = \mathbf{w}_3 \underline{\beta}_3$$
;

where

- -n is the sample size.
- $-\underline{\lambda}_k = (\lambda_{k1}, \lambda_{k2}, \dots, \lambda_{kn})^T$  for k = 1, 2, 3 are vectors of length n with the estimated lambda for each observation.
- $-\mathbf{w}_1, \mathbf{w}_2$  are  $n \times p$  data matrices containing explanatory variables for  $\lambda_1$  and  $\lambda_2$ .
- $-\mathbf{w_3}$  is a  $n \times p_2$  data matrix containing explanatory variables for  $\lambda_3$ .
- $-\underline{\beta}_1, \underline{\beta}_2, \underline{\beta}_3$  are parameter vectors used in the linear predictors of  $\lambda_1, \lambda_2$  and  $\lambda_3$ .

#### Usage

```
lm.bp( 11, 12, 1112=NULL, 13=~1, data, common.intercept=FALSE,
zeroL3=FALSE, maxit=300, pres=1e-8, verbose=getOption("verbose") )
```

#### Arguments

11	Formula of the form " $x \sim X_1 + \ldots + X_p$ " for parameters of log $\lambda_1$ .
12	Formula of the form " $y \sim X_1 + \ldots + X_p$ " for parameters of $\log \lambda_2$ .
1112	Formula of the form " $\sim X_1 + \ldots + X_p$ " for the common parameters of $\log \lambda_1$ and $\log \lambda_2$ . If the explanatory variable is also found on 11 and/or 12 then a model using interaction type parameters is fitted (one parameter common for both predictors [main effect] and differences from this for the other predictor [interaction type effect]). Special terms of the form

	"c(X1,X2)" can be also used here. These terms imply common parameters of $\lambda_1$ and $\lambda_2$ on different variables. For example if c(x1,x2) is used then use the same beta for the effect of $x_1$ on $\log \lambda_1$ and the effect of $x_2$ on $\log \lambda_2$ . For details see example 4 - dataset ex4.ita91.
13	Formula of the form " $\sim X_1 + \ldots + X_p$ " for the parameters of log $\lambda_3$ .
data	Data frame containing the variables in the model.
common.interc	ept
	Logical function specifying whether a common intercept on $\log \lambda_1$ and $\log \lambda_2$ should be used. The default value is FALSE.
zeroL3	Logical argument controlling whether $\lambda_3$ should be set equal to zero (therefore fits a double Poisson model).
maxit	Maximum number of EM steps. Default value is 300 iterations.
pres	Precision used in stopping the EM algorithm. The algorithm stops when the relative log-likelihood difference is lower than the value of pres.
verbose	Logical argument controlling whether beta parameters will we printed while EM runs. Default value is taken equal to the value of options()\$verbose. If verbose=FALSE then only the iteration number, the loglikelihood and its relative difference from the previous iteration are printed. If verbose=TRUE then the model parameters $\beta_1$ , $\beta_2$ and $\beta_3$ are additionally printed

## Value

A list object returned with the following variables.

coefficients	Estimates of the model parameters for $\beta_1$ , $\beta_2$ and $\beta_3$ . When a factor is used then its default set of constraints is used.
fitted.values	Data frame with n lines and 2 columns containing the fitted values for x and y. For the bivariate Poisson model the fitted values are given by $\lambda_1 + \lambda_3$ and $\lambda_2 + \lambda_3$ respectively.
residuals	Data frame with n lines and 2 columns containing the residuals of the model for x and y. For the bivariate Poisson model the residual values are given by $x - E(x)$ and $y - E(y)$ respectively; where $E(x) = \lambda_1 + \lambda_3$ and $E(y) = \lambda_2 + \lambda_3$ .
beta1,beta2, b	
lambda1, lambd	Vectors $\beta_1, \beta_2$ and $\beta_3$ containing the coefficients involved in the linear predictors of $\lambda_1, \lambda_2$ and $\lambda_3$ respectively. When zeroL3=TRUE then beta3 is not calculated.
	Vectors of length $n$ containing the estimated $\lambda_1$ and $\lambda_2$ for each observation
lambda3	vector containing the values of $\lambda_3$ . If zeroL3=TRUE then lambda3 is equal to zero and is not provided.
loglikelihood	Maximized log-likelihood of the fitted model. This is given in a vector form (one value per iteration). Using this vector we can monitor the log-likelihood evolution in each EM step.
AIC, BIC	AIC and BIC of the model. Values are also provided for the double Poisson model and the saturated model.
parameters	Number of parameters.
iterations	Number of iterations.
call	Argument providing the exact calling details of the lm.bp function.

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#### References

1. Karlis, D. and Ntzoufras, I. (2005). Bivariate Poisson and Diagonal Inflated Bivariate Poisson Regression Models in R. Journal of Statistical Software (to appear).

2. Karlis, D. and Ntzoufras, I. (2003). Analysis of Sports Data Using Bivariate Poisson Models. Journal of the Royal Statistical Society, D, (Statistician), 52, 381 - 393.

#### See Also

pbivpois, simple.bp, lm.dibp.

#### Examples

```
data(ex1.sim)
# Fit Double and Bivariate Poisson models ()
# Model 2: DblPoisson(11, 12)
ex1.m2<-lm.bp(x~1 , y~1 , data=ex1.sim, zeroL3=TRUE)</pre>
# Model 3: BivPoisson(11, 12, 13); same as simple.bp(ex1.sim$x, ex1.sim$y)
ex1.m3<-lm.bp(x~1 , y~1 , data=ex1.sim)</pre>
# Model 4: DblPoisson (11=Full, 12=Full)
ex1.m4<-lm.bp(x~. , y~. , data=ex1.sim, zeroL3=TRUE)</pre>
# for models 4-11 maximum number of iterations is set to 2
# Model 5: BivPoisson(l1=full, l2=full, l3=constant)
ex1.m5<-lm.bp(x~., y~., data=ex1.sim, maxit=2)</pre>
# Model 6: DblPois(11,12)
ex1.m6<-lm.bp(x~z1 , y~z1+z5 , l1l2=~z3, data=ex1.sim, zeroL3=TRUE)
# Model 7: BivPois(11,12,13=constant)
ex1.m7<-lm.bp(x~z1 , y~z1+z5 , l1l2=~z3, data=ex1.sim, maxit=2)
# Model 8: BivPoisson(11=full, 12=full, 13=full)
ex1.m8<-lm.bp(x<sup>~</sup>. , y<sup>~</sup>. , 13=<sup>~</sup>., data=ex1.sim, maxit=2)
# Model 9: BivPoisson(11=full, 12=full, 13=z1+z2+z3+z4)
ex1.m9<-lm.bp(x~., y~., 13=~.-z5, data=ex1.sim, maxit=2)
# Model 10: BivPoisson(11, 12, 13=full)
ex1.m10<-lm.bp(x~z1 , y~z1+z5 , l112=~z3, l3=~., data=ex1.sim, maxit=2)
# Model 11: BivPoisson(11, 12, 13= z1+z2+z3+z4)
ex1.m11<-lm.bp(x<sup>z</sup>1, y<sup>z</sup>1+z<sup>5</sup>, l112=<sup>z</sup>3, l3=<sup>..z5</sup>, data=ex1.sim, maxit=2)
```

General Diagonal Inflated Bivariate Poisson Model

#### lm.dibp

## Description

Produces a "list" object which gives details regarding the fit of a bivariate diagonal inflated Poisson regression model of the form

$$\begin{aligned} (X_i, Y_i) &\sim DIBP(\lambda_{1i}, \lambda_{2i}, \lambda_{3i}, D(\theta)) \text{ which is equivalent to} \\ (X_i, Y_i) &\sim (1-p)BP(x_i, y_i | \lambda_{1i}, \lambda_{2i}, \lambda_{3i}) \text{ if } x_i \neq y_i \\ (X_i, Y_i) &\sim (1-p)BP(x_i, y_i | \lambda_{1i}, \lambda_{2i}, \lambda_{3i}) + pD(x_i | \theta) \text{ if } x_i = y_i \text{ for } i = 1, 2, \dots, n \end{aligned}$$

with

$$\log \underline{\lambda}_1 = \mathbf{w}_1 \underline{\beta}, \ \log \underline{\lambda}_2 = \mathbf{w}_2 \underline{\beta} \text{ and } \log \underline{\lambda}_3 = \mathbf{w}_3 \underline{\beta}_3 ;$$

where

- -n is the sample size.
- $-\underline{\lambda}_k = (\lambda_{k1}, \lambda_{k2}, \dots, \lambda_{kn})^T$  for k = 1, 2, 3 are vectors of length *n* containing the estimated lambda for each observation.
- $-\mathbf{w}_1, \mathbf{w}_2$  are  $n \times p$  data matrices containing explanatory variables for  $\lambda_1$  and  $\lambda_2$ .
- $-\mathbf{w}_3$  are  $n \times p_2$  data matrix containing explanatory variables for  $\lambda_3$ .
- $\frac{\beta}{p}$  is a vector of length p which is common for  $\lambda_1$  and  $\lambda_2$  in order to allow for common effects.
- $-\underline{\beta}_3$  vector of length  $p_2$ .
- $-D(\theta)$  is a discrete distribution with parameter vector  $\theta$  used to inflate the diagonal.
- -p is the mixing proportion.

## Usage

```
lm.dibp( 11, 12, 1112=NULL, 13=~1, data, common.intercept=FALSE,
        zeroL3 = FALSE, distribution = "discrete", jmax = 2, maxit = 300,
        pres = 1e-08, verbose=getOption("verbose") )
```

#### Arguments

11	Formula of the form " $x \sim X_1 + \ldots + X_p$ " for parameters of $\log \lambda_1$ .
12	Formula of the form " $y \sim X_1 + \ldots + X_p$ " for parameters of $\log \lambda_2$ .
1112	Formula of the form "~ $X_1 + \ldots + X_p$ " for the common parameters of $\log \lambda_1$ and $\log \lambda_2$ . If the explanatory variable is also found on 11 and/or 12 then a model using interaction type parameters is fitted (one parameter common for both predictors [main effect] and differences from this for the other predictor [interaction type effect]). Special terms of the form "c(X1,X2)" can be also used here. These terms imply common parameters of $\lambda_1$ and $\lambda_2$ on different variables. For example if c(x1,x2) is used then use the same beta for the effect of $x_1$ on $\log \lambda_1$ and the effect of $x_2$ on $\log \lambda_2$ . For details see example 4 - dataset ex4.ita91.
13	Formula of the form " $\sim X_1 + \ldots + X_p$ " for the parameters of log $\lambda_3$ .
data	Data frame containing the variables in the model.
common.intercept	
	Logical function specifying whether a common intercept on $\log \lambda_1$ and $\log \lambda_2$ should be used. The default value is FALSE.
zeroL3	Logical argument controlling whether $\lambda_3$ should be set equal to zero

(therefore fits a double Poisson model).

distribution	Specifies the type of inflated distribution; ="discrete": Discrete(J=jmax), ="poisson" : Poisson( $\theta$ ) ="geometric": Geometric( $\theta$ ).
jmax	Number of parameters used in <i>Discrete</i> distribution. This argument is not used for the Poisson or the Geometric distributions are used as for the inflation of the diagonal.
maxit	Maximum number of EM steps. Default value is 300 iterations.
pres	Precision used in stopping the EM algorithm. The algorithm stops when the relative log-likelihood difference is lower than the value of pres.
verbose	Logical argument controlling whether beta parameters will we printed while EM runs. Default value is taken equal to the value of options()\$verbose. If verbose=FALSE then only the iteration number, the loglikelihood and its relative difference from the previous iteration are printed. If verbose=TRUE then the model parameters $\beta_1$ , $\beta_2$ and $\beta_3$ are additionally printed

## Value

A list object returned with the following variables.

v	<u> </u>
coefficients	Estimates of the model parameters for $\beta_1$ , $\beta_2$ and $\beta_3$ , $p$ and $\theta$ .
fitted.values	Data frame with $n$ lines and 2 columns containing the fitted values for $x$ and $y$ .
residuals	Data frame with $n$ lines and 2 columns containing the residuals of the model for $x$ and $y$ .
residuals	Data frame with n lines and 2 columns containing the residuals of the model for x and y given by $x - E(x)$ and $y - E(y)$ respectively; where $E(x)$ and $E(y)$ are given by the fitted.values.
beta1,beta2, b	eta3
	Vectors $\beta_1, \beta_2$ and $\beta_3$ containing the coefficients involved in the linear predictors of $\lambda_1, \lambda_2$ and $\lambda_3$ respectively. When zeroL3=TRUE then beta3 is not calculated.
lambda1, lambd	
	Vectors of length $n$ containing the estimated $\lambda_1$ and $\lambda_2$ for each observation
lambda3	vector containing the values of $\lambda_3$ . If zeroL3=TRUE then lambda3 is equal to zero and is not provided.
loglikelihood	Maximized log-likelihood of the fitted model. This is given in a vector form (one value per iteration). Using this vector we can monitor the log-likelihood evolution in each EM step.
AIC, BIC	AIC and BIC of the model. Values are also provided for the double Poisson model and the saturated model.
diagonal.distr	ibution
	label for the diagonal inflated distribution used.
р	mixing proportion.
theta	Parameter vector of the diagonal distribution. For discrete distribu- tion theta has length equal to jmax with $\theta_i = \texttt{theta[i]}$ and $\theta_0 = 1 - \sum_{i=1}^{JMAX} \theta_i$ ; for the Poisson distribution theta is the mean; for the Geo- metric distribution theta is the success probability.
parameters	Number of parameters.
iterations	Number of iterations.
call	Argument providing the exact calling details of the lm.dibp function.

#### lm.dibp

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## References

1. Karlis, D. and Ntzoufras, I. (2005). Bivariate Poisson and Diagonal Inflated Bivariate Poisson Regression Models in R. Journal of Statistical Software (to appear).

2. Karlis, D. and Ntzoufras, I. (2003). Analysis of Sports Data Using Bivariate Poisson Models. Journal of the Royal Statistical Society, D, (Statistician), 52, 381 - 393.

## See Also

pbivpois, simple.bp, lm.bp.

#### Examples

```
data(ex2.sim)
# Model 1: BivPois
ex2.m1<-lm.bp( x~z1 , y~z1+z5, l1l2=~z3, l3=~.-z5, data=ex2.sim )
# Model 2: Zero Inflated BivPois
ex2.m2<-lm.dibp( x~z1 , y~z1+z5, l112=~z3, l3=~.-z5, data=ex2.sim , jmax=0)
# for models 3-10, the maximum number of iterations is set to 2
# Model 3: Diagonal Inflated BivPois with DISCRETE(1) diagonal distribution
ex2.m3<-lm.dibp( x<sup>z</sup>1 , y<sup>z</sup>1+z5, l112=<sup>z</sup>3, l3=<sup>-</sup>.-z5, data=ex2.sim , jmax=1, maxit=2)
# Model 4: Diagonal Inflated BivPois with DISCRETE(2) diagonal distribution
ex2.m4<-lm.dibp( x<sup>z</sup>1 , y<sup>z</sup>1+z5, l112=<sup>z</sup>3, l3=<sup>-.-z5</sup>, data=ex2.sim , jmax=2, maxit=2)
# Model 5: Diagonal Inflated BivPois with DISCRETE(3) diagonal distribution
ex2.m5<-lm.dibp( x<sup>z</sup>1 , y<sup>z</sup>1+z5, l112=<sup>z</sup>23, l3=<sup>-</sup>.-z5, data=ex2.sim , jmax=3, maxit=2)
# Model 6: Diagonal Inflated BivPois with DISCRETE(4) diagonal distribution
ex2.m6<-lm.dibp( x~z1 , y~z1+z5, l112=~z3, l3=~.-z5, data=ex2.sim , jmax=4, maxit=2)
# Model 7: Diagonal Inflated BivPois with DISCRETE(5) diagonal distribution
ex2.m7<-lm.dibp( x<sup>z</sup>1 , y<sup>z</sup>1+z5, l112=<sup>z</sup>23, l3=<sup>-</sup>.-z5, data=ex2.sim , jmax=5, maxit=2)
# Model 8: Diagonal Inflated BivPois with DISCRETE(6) diagonal distribution
ex2.m8<-lm.dibp( x~z1 , y~z1+z5, l1l2=~z3, l3=~.-z5, data=ex2.sim , jmax=6, maxit=2)
# Model 9: Diagonal Inflated BivPois with POISSON diagonal distribution
ex2.m9<-lm.dibp( x~z1 , y~z1+z5, l112=~z3, l3=~.-z5, data=ex2.sim ,
                  distribution="poisson", maxit=2)
# Model 10: Diagonal Inflated BivPois with GEOMETRIC diagonal distribution
ex2.m10<-lm.dibp( x~z1 , y~z1+z5, l112=~z3, l3=~.-z5, data=ex2.sim ,
                    distribution="geometric", maxit=2)
#
# printing parameters of model 7
ex2.m7$beta1
ex2.m7$beta2
ex2.m7$beta3
ex2.m7$p
ex2.m7$theta
# printing parameters of model 9
```

pbivpois

```
ex2.m9$beta1
ex2.m9$beta2
ex2.m9$beta3
ex2.m9$p
ex2.m9$theta
```

newnamesbeta

## Internal Function of BIVPOIS Package

#### Description

This function was made only for internal use in Bivpois package and it should not be called separately

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```
pbivpois
```

Probability Function of the Bivariate Poisson Distribution

#### Description

Returns the probability the probability (or the log) of the bivariate poisson distribution for values x and y.

#### Usage

```
pbivpois(x, y=NULL, lambda = c(1, 1, 1), log = FALSE)
```

#### Arguments

x	Matrix or Vector containing the data. If $\mathbf{x}$ is a matrix then we consider as $x$ the first column and $y$ the second column. Additional columns and $\mathbf{y}$ are ignored.
У	Vector containing the data of $y$ . It is used only if $\mathbf{x}$ is also a vector. Vectors $\mathbf{x}$ and $\mathbf{y}$ should be of equal length.
lambda	Vector (of length 3) containing values of the parameters $\lambda_1$ , $\lambda_2$ and $\lambda_3$ of the bivariate Poisson distribution.
log	Logical argument for calculating the log probability or the probability function. The default value is <b>FALSE</b> .

## Details

This function evaluates the probability function (or the log) of the bivariate Poisson distribution with parameters  $\lambda_1$ ,  $\lambda_2$  and  $\lambda_3$ . Much faster than **bivpois.table** since it avoid 'for-loops' and does not use recursive relations.

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#### simple.bp

#### Value

A vector of values of the probabilities of  $PD(\lambda_1, \lambda_2, \lambda_3)$  evaluated at (x, y) when log=FALSE or the log-probabilities of  $PD(\lambda_1, \lambda_2, \lambda_3)$  evaluated at (x, y) when log=TRUE.

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#### References

1. Karlis, D. and Ntzoufras, I. (2005). Bivariate Poisson and Diagonal Inflated Bivariate Poisson Regression Models in R. Journal of Statistical Software (to appear).

2. Karlis, D. and Ntzoufras, I. (2003). Analysis of Sports Data Using Bivariate Poisson Models. Journal of the Royal Statistical Society, D, (Statistician), 52, 381 - 393.

#### See Also

bivpois.table, simple.bp, lm.bp, lm.dibp.

#### Examples

```
# probability function of (x,y)=(3,1) for lambda_1=1, lambda_2=1, lambda_3=1
pbivpois(3, 1)
# probability function of (x,y)=(3,1) for lambda_1=3, lambda_2=1, lambda_3=1
pbivpois(3, 1, lambda=c(3,1,1))
# log-probability function of (x,y)=(3,1) for lambda_1=1, lambda_2=1, lambda_3=1
pbivpois(3, 1, lambda=c(3,1,1), log=TRUE)
#
# evaluates f(1,1), f(1,3) and f(3,1) for PD(2,1,0.2)
pbivpois( c(1,3,1), c(1,1,3), c( 2,1,0.2 ) )
# same as above
pbivpois( cbind(c(1,3,1), c(1,1,3)), lambda=c( 2,1,0.2 ) )
```

simple.bp

Simple Bivariate Poisson Model

#### Description

Produces a "list" object which gives details regarding the fit of a simple bivariate Poisson model of the form  $(X, Y) \sim BP(\lambda_1, \lambda_2, \lambda_3)$ .

## Usage

```
simple.bp(x, y, ini3 = 1, maxit = 300, pres = 1e-08)
```

#### Arguments

x	Matrix or Vector containing the data. If $\mathbf{x}$ is a matrix then we consider as $x$ the first column and $y$ the second column. Additional columns and $\mathbf{y}$ are ignored.
У	Vector containing the data of $y$ . It is used only if $x$ is also a vector. Vectors $x$ and $y$ should be of equal length.
ini3	Initial value for $\lambda_3$ .
maxit	Maximum number of EM steps.
pres	Precision used in log-likelihood improvement.

#### Details

During the run of the algorithm the following details are printed: the iteration number, lambda1, lambda2, lambda3, the log-likelihood and the relative difference of the loglikelihood.

## Value

A list object returned with the following variables.

lambda	Vector with parameter values $\lambda_1$ , $\lambda_2$ , $\lambda_3$
loglikelihood	Ìaximized log-likelihood of the fitted model. This is given in a vector form (one value per iteration).Using this we may monitor the log-likelihood improvement and how EM algorithm works.
AIC, BIC	AIC and BIC of the model. Values are also given for the double Poisson model and the saturated model.
parameters	Number of parameters.
iterations	Number of iterations.

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#### References

1. Karlis, D. and Ntzoufras, I. (2005). Bivariate Poisson and Diagonal Inflated Bivariate Poisson Regression Models in R. Journal of Statistical Software (to appear).

2. Karlis, D. and Ntzoufras, I. (2003). Analysis of Sports Data Using Bivariate Poisson Models. Journal of the Royal Statistical Society, D, (Statistician), 52, 381 - 393.

#### See Also

pbivpois, lm.bp, lm.dibp

#### splitbeta

```
Examples
    #
    # Generation of BP(1,2,3) data
    x3<-rpois(100, 3)
    x1<-rpois(100, 1)+x3
    x2<-rpois(100, 2)+x3
    # fits the model
    x<-simple.bp(x1, x2)</pre>
    #
    # Monitors parameters
    x$lambda1
    x$lambda2
    x$lambda3
    #
    # alternatively (for 10 iterations)
    x<-simple.bp( cbind(x1, x2), maxit=2 )</pre>
```

splitbeta

Internal Function of BIVPOIS Package

## Description

This function was made only for internal use in Bivpois package and it should not be called separately

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