Finding optimal strategies for substitutions in soccer using a two-scale Markov Decision Process approach

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UNIVERSITÄT BAYREUTH

Motivation

- Main Question: How to weigh risk and skills in sport games?
  - Risk: Increasing chances for reaching the own objective vs. giving possibilities to the opponent
  - Skills: Actions in sports games might not be executed as expected
- Goal: Choose best risk-level depending on skills, opponent, score, time, . . .
- Previous work: Beach volleyball
- Today: Soccer

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Soccer

- Risk-level: Given by tactical formations
  - *offensive:* Increases chances to score, but higher probability of receiving a goal
  - *defensive:* Increases chances to keep the score, but lower probability of scoring
- Idea: Choose/Change risk-level during the game by substitutions
- Problem: Skills are hard to determine at that level

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  $\rightsquigarrow$ "58-,73-,79-minute rule"
- Silva and Swartz (2016): Incorporate strength factors for teams
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- strategic MDP: modeling risk and actions
  - observable states and actions
  - answering the main question
- gameplay MDP: modeling skills
  - refinement of the strategic MDP
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Beach volleyball vs. soccer

beach volleyball	**soccer**
discretization by hits	no obvious discretization
one hit (action) per player	multiple actions by one player
objective: reach 21 points	objective: score more goals within 90 minutes
teams are separated by the net	no separation between teams

Assumptions

Bounding state and action spaces:

- 3 possible formations
  - neutral (formation at start of the game)
  - offensiv
  - defensiv
- Just one substitution
- Game ends if one team leads by more than 5 goals

- No unintended changes in formations (injuries, red cards, etc.)
- No home advantages

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A strategic MDP for soccer

- State space
  $S := (\{0, 1, \ldots, 8\} \times \mathbb{Z} \times \{P_n, P_o, P_d\}) \cup \{Pen, End\}$
  - $(t, r, m) \in S$: time $t{\cdot}15$ minutes, score r, formation m
  - Pen: penalty shoot-out
  - End: end of the game
  - $S^{win} := \{(t, r, m) \in S | r \geq 6 \vee (t = 6 \wedge r > 0) \vee (t = 8 \wedge r > 0)\}$
  - $S^{lose} := \{(t, r, m) \in S | r \leq -6 \vee (t = 6 \wedge r < 0) \vee (t = 8 \wedge r < 0)\}$

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- Set of actions $C := \{n, o, d\}$
- Restrictions

$$A_s := U(s) := \begin{cases} \{o\} & \text{, if } m = P_o \\ \{n, o, d\} & \text{, if } m = P_n \\ \{d\} & \text{, if } m = P_d \end{cases}.$$

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- Transition probabilities
  - For $s = (t, r, m) \in S \setminus (S^{win} \cup S^{lose} \cup \{Pen, End\})$ with $s \neq (8, 0, m)$: $\mathbb{P}((t+1, r+n, P_a)|s, a) = p_a^n$
  - For $s = (8, 0, m) \in S$: $\mathbb{P}(Pen|s, a) = 1$
  - For $s \in (S^{win} \cup S^{lose} \cup \{Pen, End\})$: $\mathbb{P}(End|s, a) = 1$
- Costs

$$c(s, a, s') = \begin{cases} -1 & \text{, if } s' \in S^{win} \\ -\frac{1}{2} & \text{, if } s' = Pen \\ 0 & \text{, else} \end{cases}$$

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Finding optimal strategies

Lemma

Let $J^*(s)$ denote the optimal cost function. Then:

$$J^*(t, r, m) \geq J^*(t, r', m) \ \ \forall \ r' \geq r.$$

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Let $s = (t, r, m) \in S \setminus (S^{win} \cup S^{lose} \cup \{Pen, End\})$ with $n \in A_s$ und $s \neq (8, 0, m)$. Then:

$$J^*(s) = \min \left\{ \sum_{k \in \mathbb{N}} p_n^k J^*(s), J_o(s), J_d(s) \right\}$$

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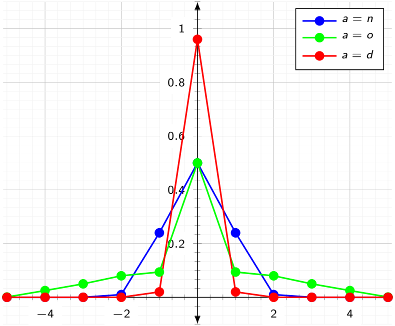

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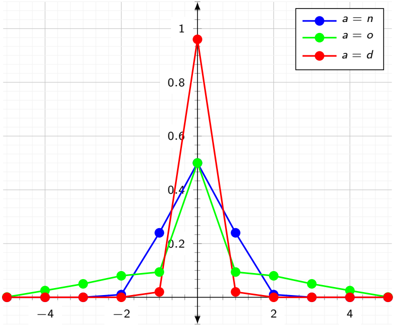

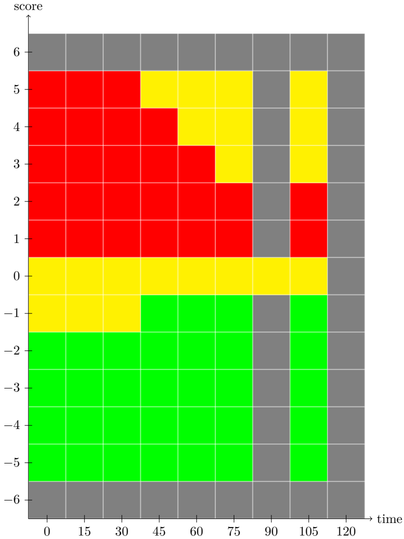

A gameplay MDP for soccer

- How to get p_a^n? $\rightsquigarrow$ gameplay MDP
- Model soccer game in detail and incorporate strategic decisions
- Ideas:
  - *States:* Player, who is in possession of the ball + goal states
  - *Actions:* Soccer actions that change ball possession, e. g. passing, crossing, shooting, saving
  - *Restrictions:* Only one step towards the goal; Actions depending on position
  - *Transition probabilities:* Depend on players' skills and team formations

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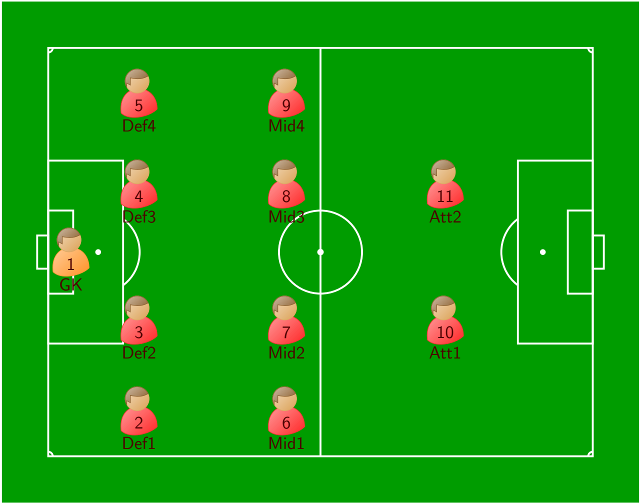

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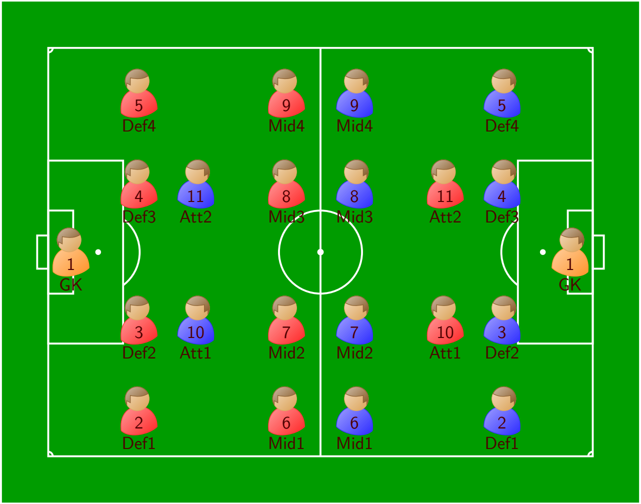

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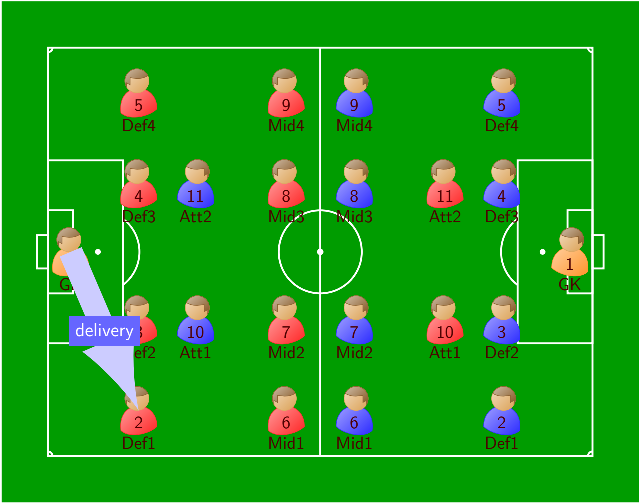

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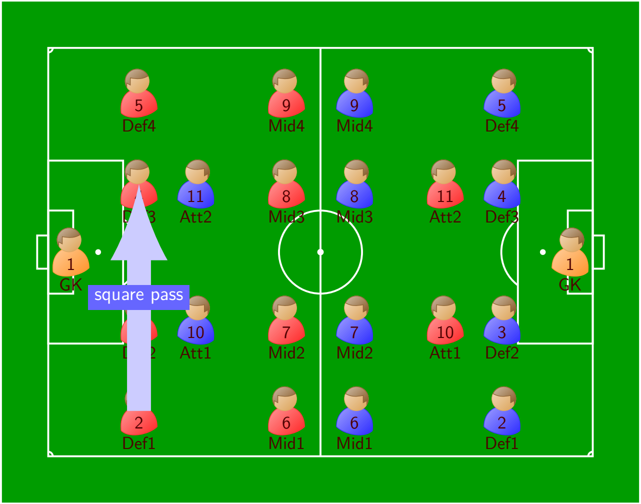

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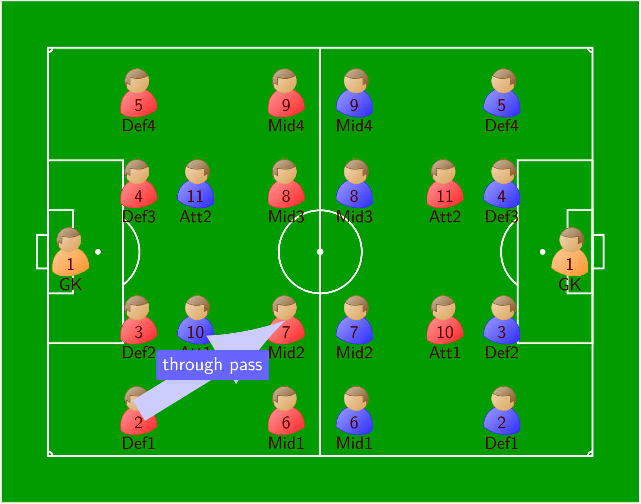

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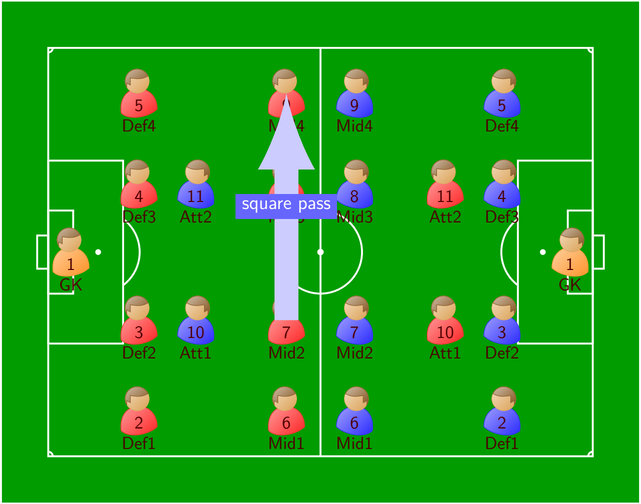

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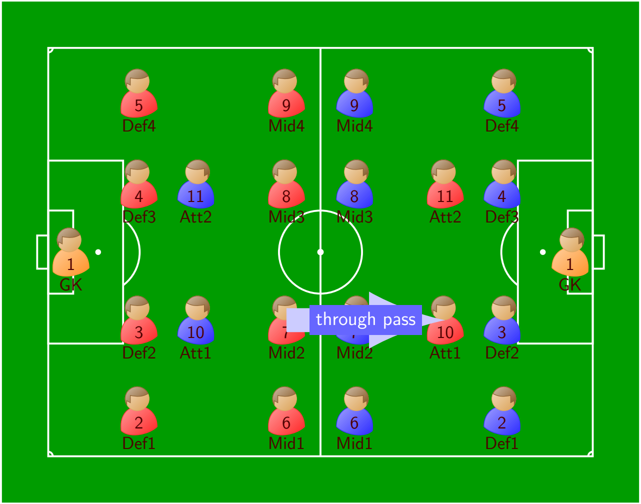

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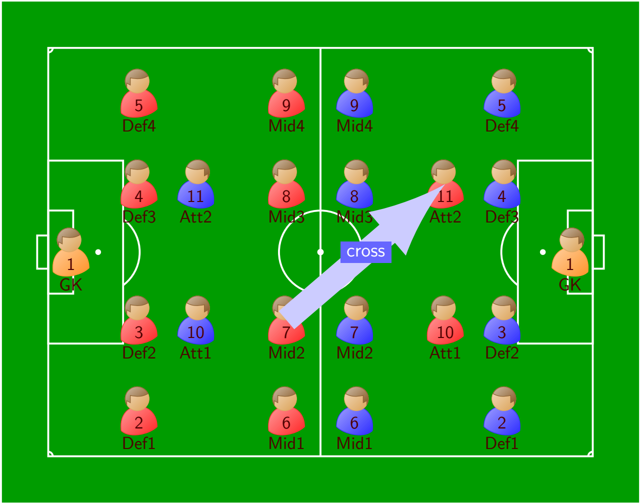

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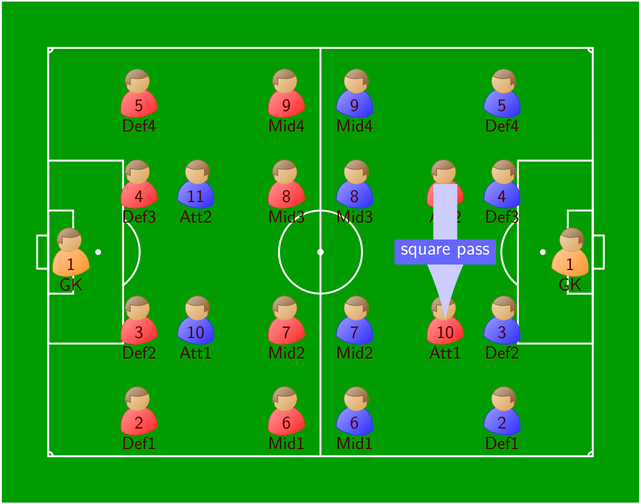

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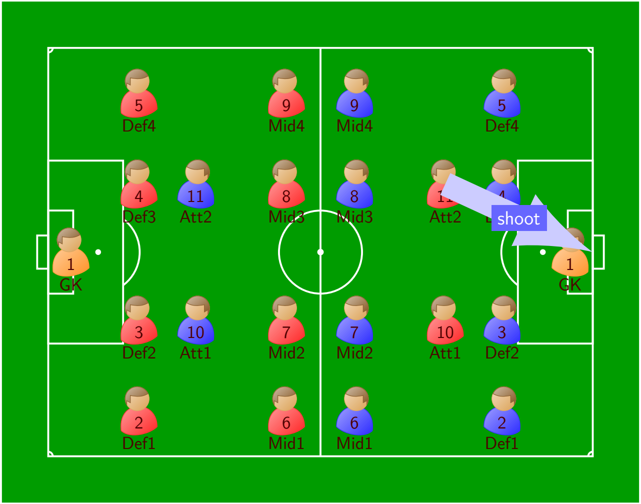

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- Players' abilities given by
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  For $s \in Def_P \cup Mid_P \cup Att_P$ and $a = (squarepass, sp) \in A_s$:

$$\mathbb{P}(sp|s,a) = p_s^{squarepass} \text{ and}$$

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Gameplay MDP – example (artificial; randomized)

No.	Position	$p^{squarepass}$	$p^{throughpass}$	p^{cross}	p^{shot}	p^{save}	*height*
1	GK	0.919170	0.624955	0.724972	0.615675	0.815179	1.668144
2	Def	0.969440	0.628735	0.407385	0.409849	0.804164	1.711843
3	Def	0.865168	0.610413	0.517610	0.421495	0.569627	1.902436
4	Def	0.884845	0.736387	0.778947	0.564421	0.694788	1.860544
5	Def	0.860315	0.726746	0.614380	0.441174	0.726783	1.732182
6	Mid	0.975816	0.753700	0.675745	0.653772	0.641041	1.702318
7	Mid	0.867531	0.674613	0.776500	0.285493	0.798201	1.770497
8	Mid	0.910666	0.789258	0.786211	0.289292	0.755130	1.605349
9	Mid	0.840645	0.645869	0.604986	0.293282	0.849243	1.766503
10	Att	0.902109	0.729633	0.575996	0.454477	0.562089	1.771343
11	Att	0.873020	0.893718	0.440173	0.459758	0.640865	1.950173

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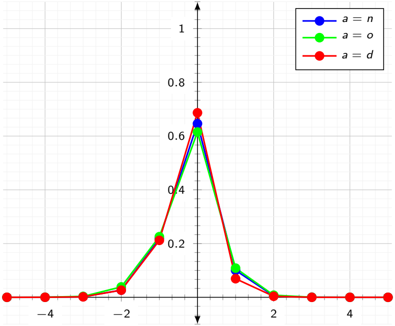

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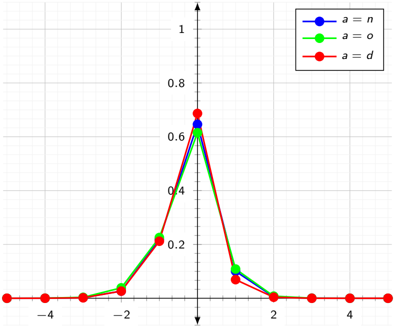
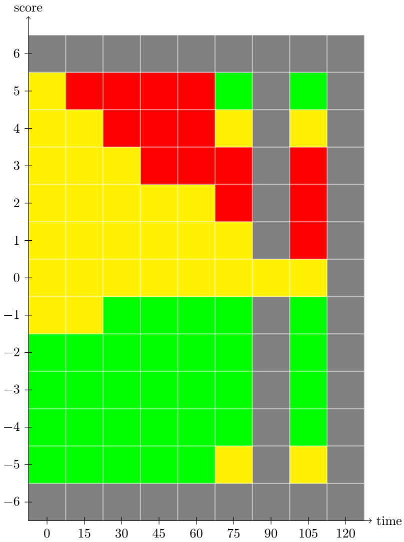
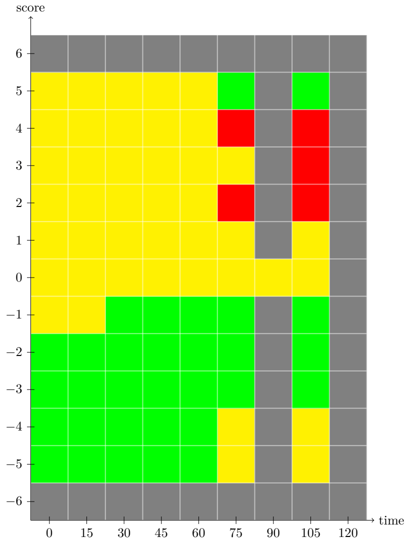




Other examples randomized



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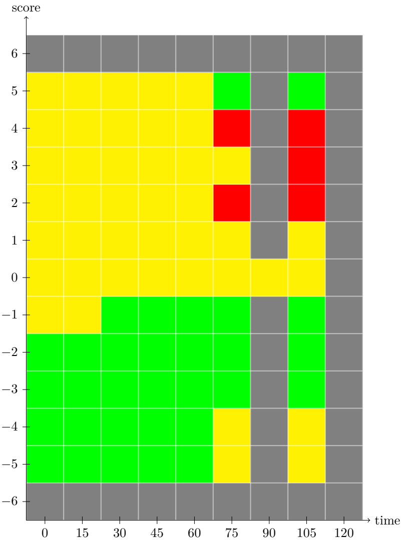

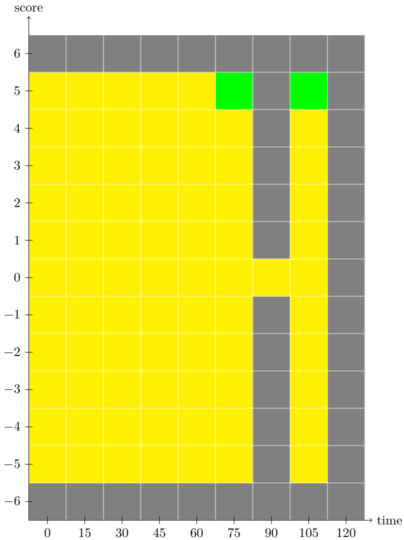

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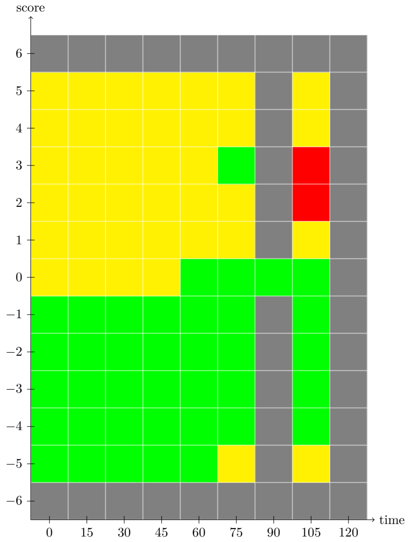

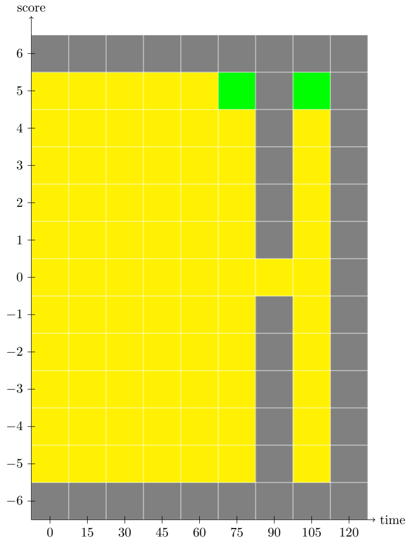

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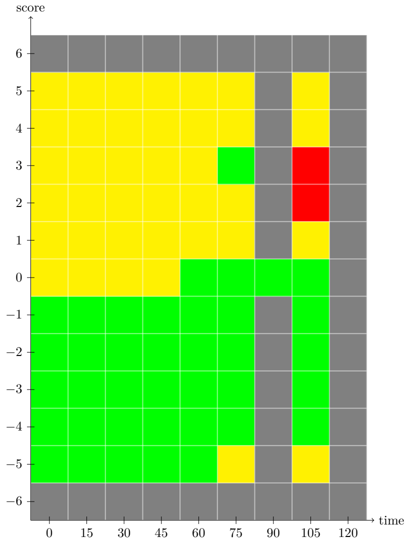

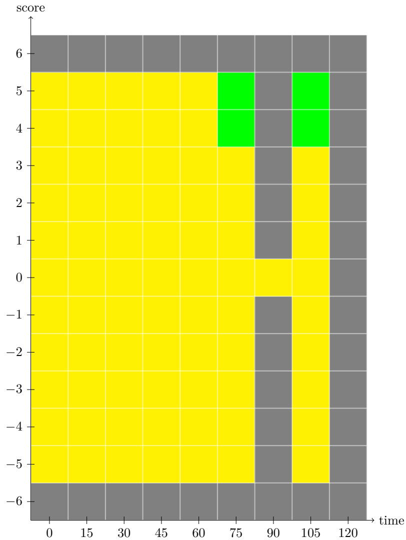

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  - Simple s-MDP for strategic substitution in soccer
  - First detailed g-MDP for simulating s-MDP probabilities
  - Working example on artificial data
- Future work:
  - Expand s-MDP
  - Enhance g-MDP with more data and more game dynamics
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Finding optimal strategies for substitutions in soccer using a two-scale Markov Decision Process approach

Jörg Rambau
Rónán Richter (speaker)

MathSport International Conference

Athens, July 2019

UNIVERSITÄT BAYREUTH

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