Rest Differences Among Teams in European Football Leagues

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Rest Difference

- Opposing teams may not have had equal rest periods after their last game in the league.

- Rest difference refers to the difference in rest durations of two opposing teams between consecutive games.

Timetable Fairness Criteria

- Time-constrained tournaments
  - Breaks
  - Carryover effect
- Time-relaxed tournaments
  - Travel distance
  - Rest time between consecutive games
  - Number of games played at any given time

From comments by team managers, coaches and players in the popular media, we believe that rest differences among opposing teams should also be paid attention to as a fairness criterion.

Rest Mismatch

- A Rest Mismatch occurs whenever there is a difference in the rest periods of opposing teams regardless of the duration of that difference.
  - T. Atan and B. Cavdaroglu. Minimization of rest mismatches in round robin tournaments. *Computers and Operations Research*, 99:78-89, 2018.

Practice

- Many football leagues in Europe are organized in round-robin format.

- The Turkish Süperlig has 18 teams.
  - Mirrored double round robin tournament
  - 17 + 17 = 34 rounds
  - 9 games in each round
  - Canonical schedule

Round   7 vs 8
Round   6 vs 7

- A league schedule is announced before the seaso

- The game periods (days of games) in each round are announced as the league progresses.

Problem

- Determine the game periods of each game <u>for a given schedule</u> to minimize the total rest difference

- For each round, the number of game periods and the number of games in each period are predetermined.

Notation

- *n* : Number of teams (even)
- *p* : Number of consecutive periods in each round
- Sets
  - *P* : Periods, *k* = 1, …, *p*; *R* : Rounds, *r* = 1, …, 2(*n*-1); *T* : Teams, *i* = 1, …, *n*.
- Parameters
  - λ_r : The number of periods between the first periods of round *r* and *r-1*
  - $nGames_{r,k}$: The number of games in period *k* of round *r*
  - $play_{i,j,r}$ = 1 if teams *i* and *j* play against each other in round *r*; 0 otherwise
- Decision variables
  - $p_{i,r}$: The number of periods team *i* rested less than its competitor in round *r*
  - $x_{i,r,k}$ = 1 if team *i* has its game in period *k* of round *r*; 0 otherwise
  - *z* : Sum of rest differences of teams

MIP Model

- (1) Minimize the sum of rest differences

subject to:

- (2) Each team must play a game in only one period in each round
- (3) Opposing teams should be assigned to the same period
- (4) The number of games in a period should be equal to a predetermined number
- (5) Rest differences of opposing teams should be equal (soft)

MIP Model

$$\min \quad z = \sum_i \sum_r p_{i,r} \tag{1}$$

subject to:

$$\sum_k x_{i,r,k} = 1 \quad \forall i \in T, \forall r \in R \tag{2}$$

$$play_{i,j,r} \cdot x_{i,r,k} = play_{i,j,r} \cdot x_{j,r,k} \quad \forall i,j \in T, \forall r \in R, \forall k \in P \tag{3}$$

$$\sum_i x_{i,r,k} = 2 \cdot nGames_{r,k} \quad \forall r \in R, \forall k \in P \tag{4}$$

$$play_{i,j,r} \cdot \left(\sum_k k \cdot x_{i,r,k} - \left(\sum_k k \cdot x_{i,r-1,k} - \lambda_r \right) + p_{i,r} \right)$$
$$= play_{i,j,r} \cdot \left(\sum_k k \cdot x_{j,r,k} - \left(\sum_k k \cdot x_{j,r-1,k} - \lambda_r \right) + p_{j,r} \right)$$
$$\forall i,j \in T, \forall r \in R \setminus \{1\} \tag{5}$$

$$x_{i,r,k} \in \{0,1\} \quad \forall i \in T, \forall r \in R, \forall k \in P \tag{6}$$

$$p_{i,r} \geq 0 \quad \forall i \in T, \forall r \in R \tag{7}$$

Minimizing Rest Mismatches

- The rest difference model can be modified slightly to minimize the number of rest mismatches.
  - Introduce a set of binary rest mismatch variables and relate them to the rest difference variables (p).

Computational Experiments

- Intel i5-4570 CPU 3.2 GHz
- 8Gb RAM
- GAMS/Gurobi

- Out of memory
  - Single round robin, 40 teams, 4 periods with 5 games in each round
  - After 7 hours

- We solved the models for several leagues in Europe for the 2017-18 season.
- League instances (16→20 teams, 2→4 periods) were relatively quick to solve.
  - Rest mismatch problems were harder, sometimes taking about half an hour to solve.

2017-18 Season

Country	Canonical (Teams)	Rest Difference	Optimal	Max Possible	Rest Mismatch	Optimal	Max Possible
Belgium	No (16)	162 (56%)	104	232 (30%)	139 (74%)	80	217 (36%)
England	No (20)	234 (86%)	126	306 (24%)	177 (121%)	80	227 (22%)
France	No (20)	230 (117%)	106	294 (22%)	202 (143%)	83	288 (30%)
Germany	No (18)	188 (71%)	110	224 (16%)	158 (86%)	85	214 (26%)
Italy	No (20)	172 (121%)	78	202 (15%)	160 (142%)	66	192 (17%)
Netherlands	No (18)	190 (94%)	98	298 (36%)	167 (109%)	80	274 (39%)
Portugal	Yes (18)	308 (77%)	174	410 (25%)	225 (91%)	118	293 (23%)
Russia	Yes (16)	200 (54%)	130	288 (31%)	156 (68%)	93	224 (30%)
Spain	Yes (20)	314 (64%)	192	484 (35%)	244 (83%)	133	364 (33%)
Turkey	Yes (18)	280 (52%)	184	394 (29%)	210 (68%)	125	297 (29%)

Note that rest differences/mismatches are also impacted by the number of days in each round and the number of games scheduled on each day.

2017-18 Season

- Rest difference
  - Between 52% and 121% off from the optimal
    - Closest: Turkey   Farthest: Italy
  - Between 15% and 36% improvement from the worst possible
    - Most improved: Netherlands
- Rest mismatch
  - Between 68% and 142% off from the optimal
    - Closest: Russia, Turkey   Farthest: Italy
  - Between 17% and 36% improvement from the worst possible
    - Most improved: Netherlands

Improvements from the worst are not by design, most schedules could probably do a lot better.

Conclusion

- League organizers do not pay much attention to rest differences.
  - Broadcast ratings
  - Security
  - Shared stadiums
- Comments by stakeholders suggest that they should.
- In congested schedules rest differences become more important.