Extreme value prediction: an application to sport records

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Outline

In this work we investigate the use of bootstrap calibration applied to extreme value theory for the prediction of sport records.

1. Extreme value theory
2. Bootstrap calibrated prediction
3. Application to athletic records

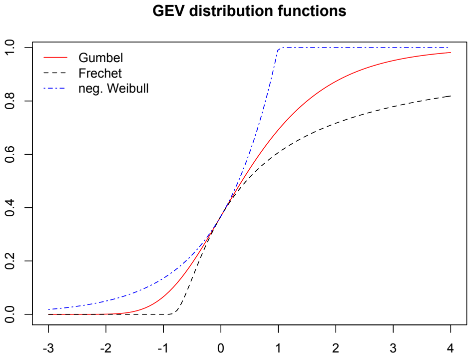
Extreme value theory

- $\{X_t\}_{t\geq 1}$ a discrete-time stochastic process
- $Y_i = \max_{k\in T_i} X_k$ the maximum of the process over time interval T_i, $i \geq 1$
- Under suitable conditions and if the number of observations in each period is big enough, the Y_i's are approximately independent and with the same generalised extreme value (GEV) distribution (Coles, 2011)

Generalised extreme value (GEV) distribution

$$G(z;\mu,\sigma,\xi) = \exp\left\{-\left[1+\xi\left(\frac{z-\mu}{\sigma}\right)\right]^{-1/\xi}\right\},$$

with z such that $1+\xi(z-\mu)/\sigma > 0$ and $\sigma > 0$.



Prediction

- $Y = (Y_1, \cdots, Y_n)$, $n > 1$ observable sequence of maxima of a process
- $Z = Y_{n+1}$ a future or not yet available observation of the maximum over the next time interval
- $Y_1, \cdots, Y_n$ and $Z = Y_{n+1}$ independent random variables with the same GEV distribution
- We want predict Z on the basis of an observed sample from Y

Prediction limits

Given an observed sample $y = (y_1, \ldots, y_n)$, an α-prediction limit for Z is a function $c_\alpha(y)$ such that

$$P_{Y,Z}\{Z \leq c_\alpha(Y); \theta\} = \alpha,$$

for every $\theta \in \Theta$ and for any fixed $\alpha \in (0, 1)$.

The α-quantile $\hat{z}_\alpha$ of the estimative distribution $G(z; \hat{\theta})$ has coverage α plus an error term that may be substantial.

Bootstrap calibration

The bootstrap-calibrated predictive distribution is

$$G_c^b(z;\hat{\theta}) = \frac{1}{B}\sum_{b=1}^{B} G\{z_\alpha(\hat{\theta}^b);\hat{\theta}\}|_{\alpha=G(z;\hat{\theta})}$$

- $\hat{\theta}$ a suitable estimator for the parameter θ
- y^b, $b = 1,\ldots,B$, parametric bootstrap samples generated from the estimative distribution of the data
- $\hat{\theta}^b$, $b = 1,\ldots,B$, the corresponding estimates

The corresponding α-quantile defines, for each $\alpha \in (0,1)$, a prediction limit having coverage probability equal to the target α (Fonseca et al. 2014).

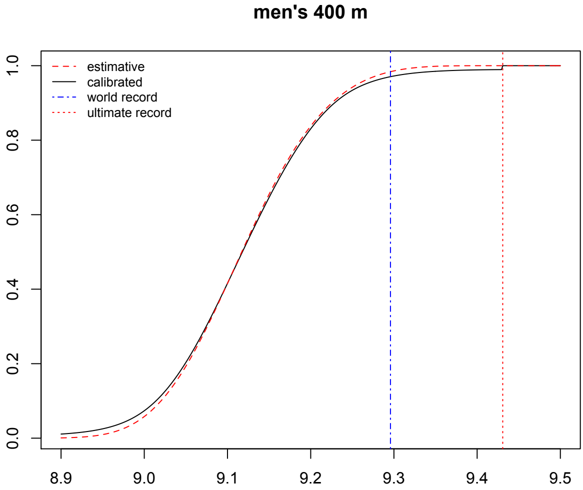
Application to athletic records

- Data from the web site of the International Association of Athletics Federations (IAAF)
- Annual records for males and females, for some athletic events, starting from 2001 (18 observations)
- Times transformed into mean speeds so that, for each event, the higher the best

Estimates for the shape parameter ξ: * means that world record is not included in the data:

gmle	100 m	200 m	400 m	10,000 m	long jump	javelin
men	-0.1281	-0.0553	-0.2246	-0.0819	-0.3104*	-0.3755*
women	-0.3069*	-0.3006*	-0.1330*	-0.1803	0.3311*	-0.1864

Men's 400 m

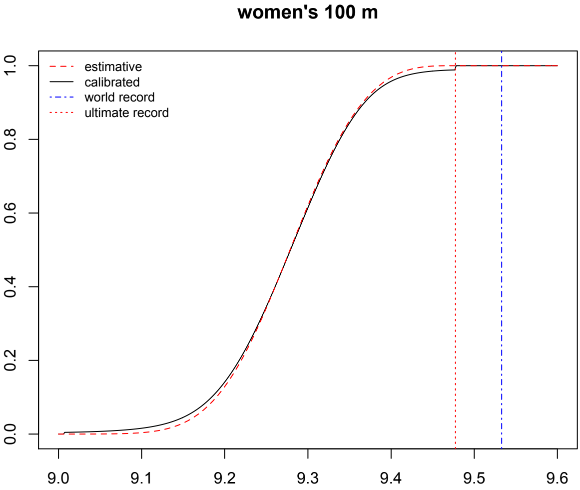

men's 400 m
estimative
calibrated
world record
ultimate record
0.0
0.2
0.4
0.6
0.8
1.0
8.9
9.0
9.1
9.2
9.3
9.4
9.5

Men's

	100 m	200 m	400 m	10,000 m	long jump	javelin
UL	10.797	12.052	9.431	6.804	8.871	95.364
α_{UL}	0.009	0.008	0.011	0.008	0.011	0.013
WR	10.438	10.422	9.296	6.339	8.95*	98.48*
α_{WR}	0.031	0.057	0.029	0.054	-	-
$\hat{\alpha}_{WR}$	0.018	0.051	0.016	0.046	-	-
T_{WR}	31.79	17.51	33.91	18.62	-	-
$\hat{T}_{WR}$	55.44	19.49	62.23	21.47	-	-

Table: Men's summary results. * means that the corresponding world record is not included in the data.

Women's 100 m


women's 100 m
estimative
calibrated
world record
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0.0
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0.8
1.0
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9.1
9.2
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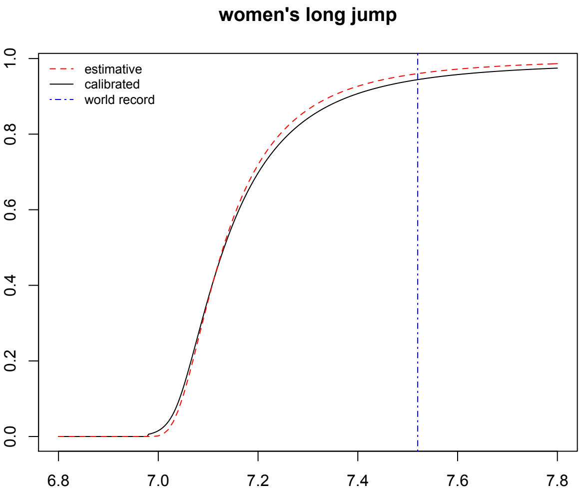
9.6

Women's long jump



Women's

	100 m	200 m	400 m	10,000 m	long jump	javelin
UL	9.477	9.368	8.449	5.897	-	78.318
α_{UL}	0.012	0.011	0.009	0.010	-	0.010
WR	9.533*	9.372*	8.403*	5.690	7.52*	72.28
α_{WR}	-	-	0.009	0.028	0.056	0.084
$\hat{\alpha}_{WR}$	-	-	0.000	0.015	0.040	0.072
T_{WR}	-	-	105.55	36.05	17.93	11.83
$\hat{T}_{WR}$	-	-	∞	68.44	25.13	13.81

Table: Women's summary results. * means that the corresponding world record is not included in the data.

Conclusions and ongoing work

We have applied the bootstrap calibration to data coming from athletic performances, achieving more accurate estimates of the probability of a new world record within the next season.

We are now working to extend calibration beyond the upper limit of the estimative distribution (non regular models).

References

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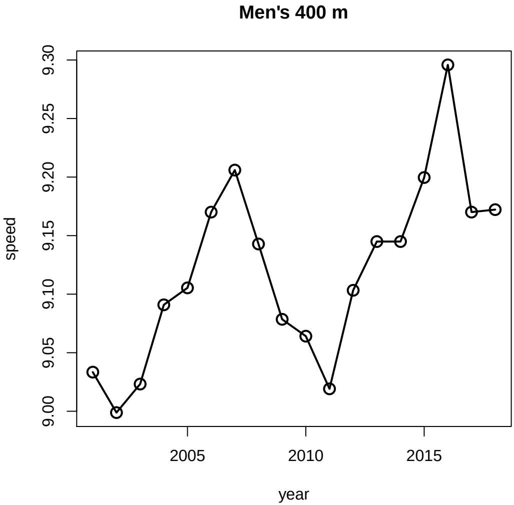
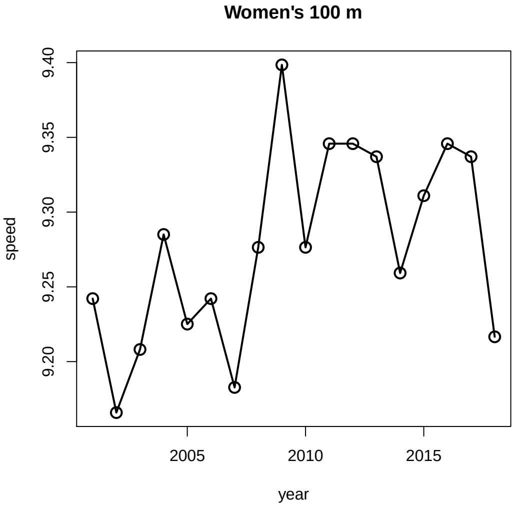
- Fonseca, G., Giummolè, F., Vidoni, P.: Calibrating predictive distributions. Journal of Statistical Computation and Simulation, **84**, 373–383 (2014).

Men's 400 m


Men's 400 m
speed
9.00
9.05
9.10
9.15
9.20
9.25
9.30
2005
2010
2015
year

Women's 100 m



Women's long jump