Will Groups of 3 Ruin the World Cup?

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Motivation

- Soccer World Cup: the most popular sporting event in the world, even more widely viewed and followed than the Olympic Games.
- Organized every 4 years by FIFA.
- 2026: 48 teams, for the first time.
- 16 **groups of 3**. Each group will play a single round-robin tournament, and the winner and runner-up will advance to the KO stage.

Using groups of 3 raises several fairness issues

Match 1	Match 2	Match 3
A–B	A–C	B–C

- Problem 1: Schedule imbalance
- **A more serious issue is the subject of this talk: the suspicion of match fixing (or collusion).**
- After Match 2, Teams B and C will know what results of Match 3 will let them advance to the KO stage. **Suspicion of collusion occurs when a result lets both of them advance, at the expense of Team A.**
- It can badly harm the tournament and more globally the game of soccer, **whether the match is actually fixed or not**, since outcome uncertainty is at the very root of sport's popularity.
- **"Disgrace of Gijón"** (1982): most famous example of match fixing in the history of soccer. West Germany 1-0 Austria, both teams qualified at the expense of Algeria who had played the day before.
- To prevent this to happen again, FIFA decided that all teams in a given group would play their last group match at the same time. But with groups of 3??

Risk of collusion already exists in groups of 4

- Even in groups of 4, playing the last two group games at the exact same time does not fully prevent collusion.
- Denmark-France (0-0 on June 26, 2018 during the 2018 FIFA World Cup) is a recent example of tacit collusion in this context. Both teams knew that a draw would let them both advance to the KO stage whatever the result of Australia-Peru.
- Denmark's manager Åge Hareide said after the game: "We just needed one point, we were up against one of the best teams in the world at counterattacks, so we would have been stupid to open up a lot of space. We stood back and got the result we needed, it was a 0-0 and we're very pleased with that".
- Denmark-Sweden (2-2) at UEFA Euro 2004 is another example of a tacit collusion situation: a 2-2 tie would qualify both teams at the expense of Italy, whatever the result of Italy against Bulgaria.
- **Risk of collusion will be worse in groups of 3. Our goal: quantify it.**

Objectives

- **Quantify the risk of suspicion of match fixing** in groups of 3, when 2 teams advance to the next phase.
- Quantify **impact of match schedule** on the risk of collusion.
- Quantify **impact of competitive balance** on the risk of collusion.
- Quantify **impact of point system**: 3-1-0 vs alternate point systems that forbid draws (3-0 and 3-2-1-0).
- **Suggest alternate formats** for a 48 team World Cup that would decrease or even eliminate the risk of collusion.

Occurrences of suspicion of match fixing

Assumptions:

- 3-1-0 point system.
- Tie-breaking rules:
  1. Overall goal difference
  2. Overall goals scored
  3. For the purpose of this study we only need to consider the further criterion: if exactly 2 teams are still even after criteria (1) and (2) are applied, the winner (if any) of the match between these two teams is ranked higher.

We say that the suspicion of match fixing is **aggravated** when Team B or C can win the group even after losing its last game.

Possible situations after Match 2

1. **Team A has two wins.** A has 6 pts, advances to KO stage.
2. **Team A has one win and one draw.** A has 4 pts, advances to KO stage.
3. **Team A has one win and one loss.**
   1. GD$_A > 0$: A has won against, say, B with a margin of m goals, and lost to C with a margin of n goals, with GD$_A = m - n > 0$. If B does not win against C, then it does not advance to the next round. If B wins against C, let p be the goal margin of this win. Then A, B, and C all have 3 pts, B and C have goal differences $p - m$ and $n - p$, one of which is strictly smaller than $m - n$. No match fixing can eliminate A.
   2. GD$_A < 0$: GD$_A = m - n < 0$. If B wins m - 0 against C, then A, B, and C all have 3 pts, B has goal difference 0, C has goal difference $n - m > 0$, both better than the goal difference of A, and the final group ranking is C $>$ B $>$ A. **Aggravated SMF**.
   3. GD$_A = 0$: If B wins $(m + k)$ - k against C with k large enough, then A, B, and C all have 3 pts and goal difference 0, but B and C will finish ahead of A thanks to a larger number of goals scored. **SMF**.
4. **Team A has two draws**. Then if B and C draw k - k with k large enough, B and C will eliminate A thanks to a larger number of goals scored. **SMF**.
5. **Team A has one draw and one loss.** Then if B and C draw they will have 2 and 4 pts and will eliminate A (1 pt). **SMF**.
6. **Team A has two losses.** Then Team A has zero pt, already eliminated.

The corresponding probabilities

Win prob.	A	B	C
A		p_{AB}	p_{AC}
B	p_{BA}		p_{BC}
C	p_{CA}	p_{CB}	

Table: Win probabilities: p_{XY} is the probability that Team X wins against Team Y; $d_{XY} := 1 - p_{XY} - p_{YX}$. $p_{<0}$ (resp. p_0, $p_{>0}$) = probability that A has negative (resp. null, positive) goal difference, i.e., $\text{GD}_A < 0$ (resp. $= 0$, > 0) *given that A has one win and one loss in the group stage*

Situation of Team A after Match 2	Probability	SMF	SMF*
Two wins	$p_{AB}p_{AC}$		
One win and one draw	$p_{AB}d_{AC} + d_{AB}p_{AC}$		
One win and one loss, $\text{GD}_A > 0$	$p_{>0}(p_{AB}p_{CA} + p_{BA}p_{AC})$		
One win and one loss, $\text{GD}_A = 0$	$p_0(p_{AB}p_{CA} + p_{BA}p_{AC})$	✔	
One win and one loss, $\text{GD}_A < 0$	$p_{<0}(p_{AB}p_{CA} + p_{BA}p_{AC})$	✔	✔
Two draws	$d_{AB}d_{AC}$	✔	
One draw and one loss	$p_{BA}d_{AC} + d_{AB}p_{CA}$	✔	
Two losses	$p_{BA}p_{CA}$		

Probability of suspicion of match fixing

Proposition

The probability of suspicion of match fixing in a given group of 3 is

$$p_{\mathrm{SMF}} := d_{AB}p_{CA} + p_{BA}d_{AC} + d_{AB}d_{AC} + p_{\leq 0}\left(p_{AB}p_{CA} + p_{BA}p_{AC}\right). \quad (1)$$

The probability of aggravated suspicion of match fixing in a given group of 3 is

$$p^{*}_{\mathrm{SMF}} := p_{<0}\left(p_{AB}p_{CA} + p_{BA}p_{AC}\right). \quad (2)$$

- Assume perfect competitive balance: $p := p_{AB} = p_{BA} = p_{AC} = p_{CA} = p_{BC} = p_{CB} \leq \frac{1}{2}$. Then $d_{AB} = d_{AC} = d_{BC} = 1 - 2p$ and

$$p_{\mathrm{SMF}} = 2p(1-2p) + (1-2p)^2 + 2p_{\leq 0}p^2 = 1 - 2p + 2p_{\leq 0}p^2$$

- When $p = \frac{1}{3}$, $p_{\mathrm{SMF}} = \frac{1}{3} + \frac{2}{9}p_{\leq 0}$. Assuming $p_{\leq 0} = 0.6$, we get $p_{\mathrm{SMF}} = \frac{7}{15}$.
- When $p = \frac{3}{8}$, $p_{\mathrm{SMF}} = \frac{1}{4} + \frac{9}{32}p_{\leq 0} = \frac{67}{160} \approx 42\%$.
- Both values are very close to 50%! **In the situation of perfect competitive balance, the risk of suspicion of match fixing is very high.**

Probability of suspicion of match fixing

Corollary

The probability of suspicion of match fixing is maximum, equal to 1, in the case where $d_{AB} = d_{AC} = 1$.

- This corollary somewhat explains why FIFA has considered banning draws during the group stage. All group stage matches would have a winner and a loser, possibly decided by a penalty shootout.
- In this case

$$p_{\mathrm{SMF}} = p_{\leq 0}\left(p_{AB}p_{CA} + p_{BA}p_{AC}\right), \qquad p^*_{\mathrm{SMF}} = p_{<0}\left(p_{AB}p_{CA} + p_{BA}p_{AC}\right).$$

- However, the values of p_{AB}, p_{BA}, p_{AC}, p_{CA} are inflated, compared with the case where draws are allowed, since the probability of a draw between Teams X and Y is redistributed to both win probabilities p_{XY} and p_{YX}.
- For instance, if we assume perfect competitive balance, then $p_{\mathrm{SMF}} = p_{\leq 0}/2$ is typically $\geq \frac{1}{4}$, while $p^*_{\mathrm{SMF}} = p_{<0}/2 \approx \frac{1}{4}$.
- **Forbidding draws does not eliminate the risk of collusion**. The situations where A has one win and one loss and a nonpositive goal difference will still be prone to match fixing.

Probability of suspicion of match fixing

Corollary

The probability of suspicion of match fixing is minimum, equal to 0, if and only if one of those 3 conditions holds:

(i) $p_{AB} = 1$ *and* ($p_{CA} = 0$ *or* $p_{\leq 0} = 0$)*: A surely wins against B, and it cannot lose against C, or if it loses against C its global goal difference* GD_A *can only be positive.*

(ii) *Same with B* $\longleftrightarrow$ *C.*

(iii) $p_{BA} = p_{CA} = 1$*: A surely loses against B and C.*

- To minimize probability of SMF, Team A should be the *a priori* strongest team in the group (so it is close to satisfy one of the first 2 conditions above) or the *a priori* weakest team in the group, if very weak (so it is close to satisfy the last condition above). Team A should not be the middle team.
- However, conditions (i), (ii), or (iii) are never satisfied in practice: there is always a positive probability that an underdog draws or wins, even if it is small. **Suspicion of match fixing cannot be avoided**.

Probability of suspicion of match fixing

Corollary

Assume one of the following conditions:

(i) *All the probabilities p_{AB}, p_{BA}, p_{AC}, p_{CA}, $p_{\leq 0}$ are strictly positive.*

(ii) *The probabilities d_{AB} and d_{AC} are strictly positive.*

Then the risk of collusion cannot be avoided: $p_{\mathrm{SMF}} > 0$.

Probability of suspicion of match fixing in at least one of the 16 groups

Proposition

Let us assume that the same values of p_{AB}, p_{BA}, p_{AC}, p_{CA}, $p_{<0}$, and $p_{\leq 0}$ apply to all 16 groups of the World Cup, and that the results in the 16 groups are all independent. Let N_{SMF} (resp. N^*_{SMF}) be the number of groups in which SMF (resp. aggravated SMF) occurs. Then for all $k \in \{0, 1, \ldots, 16\}$,

$$\begin{aligned} \mathbb{P}(N_{\mathrm{SMF}} = k) &= \frac{16!}{k!(16-k)!} p_{\mathrm{SMF}}^k (1 - p_{\mathrm{SMF}})^{16-k} \\ \mathbb{P}(N^*_{\mathrm{SMF}} = k) &= \frac{16!}{k!(16-k)!} (p^*_{\mathrm{SMF}})^k (1 - p^*_{\mathrm{SMF}})^{16-k}. \end{aligned}$$

In particular, the probability that there is SMF for at least one group is

$$p_{\mathrm{SMF}}(16) = 1 - (1 - p_{\mathrm{SMF}})^{16}, \quad p^*_{\mathrm{SMF}}(16) = 1 - (1 - p^*_{\mathrm{SMF}})^{16}.$$

There are on average

$$\mathbb{E}[N_{\mathrm{SMF}}] = 16\, p_{\mathrm{SMF}} \qquad \textit{(resp. } \mathbb{E}[N^*_{\mathrm{SMF}}] = 16\, p^*_{\mathrm{SMF}}\textit{)}$$

groups in which SMF (resp. aggravated SMF) occurs.

Impact of match schedule

Win prob.	S	M	W
S (Strong)		$p_{SM} = 50\%$	$p_{SW} = 80\%$
M (Middle)	$p_{MS} = 20\%$		$p_{MW} = 50\%$
W (Weak)	$p_{WS} = 5\%$	$p_{WM} = 20\%$	

Table: Win probabilities: p_{XY} is the probability that Team X wins against Team Y.

A	S	M	W
$p_{\leq 0}$	30%	60%	90%
$p_{<0}$	10%	40%	80%
p_{SMF}	14.6%	47.4%	52.7%
p^*_{SMF}	1.9%	11.6%	14.8%
$p_{\mathrm{SMF}}(16)$	91.9%	99.997%	99.999%
$p^*_{\mathrm{SMF}}(16)$	25.8%	86.1%	92.3%
$\mathbb{E}[N_{\mathrm{SMF}}]$	2.3	7.6	8.4
$\mathbb{E}[N^*_{\mathrm{SMF}}]$	0.3	1.9	2.4

Impact of match schedule

- For this plausible example, it is apparent that in order to minimize the risk of collusion, **Team A should be the a priori strongest team in the group**.
- Indeed, if Team A is the *a priori* strongest in the group, it would likely be already qualified after Match 2.
- If this schedule (A = S) is implemented:
  - The *a priori* strongest team in the group, if it is not already qualified after Match 2, might be the victim of a collusion between the two other teams.
  - The *a priori* strongest team in the group, if it advances to the KO stage, will enjoy more rest days than the other qualified team before the round of 32.
  - In all groups, the third match will oppose the two *a priori* weakest teams in the group.

Impact of competitive balance

	Perfect balance			Imbalance			Strong imbalance		
Win prob.	S	M	W	S	M	W	S	M	W
S (Strong)		37.5%	37.5%		50%	80%		70%	90%
M (Middle)	37.5%		37.5%	20%		50%	10%		70%
W (Weak)	37.5%	37.5%		5%	20%		2%	10%	

Table: Win probabilities: p_{XY} is the probability that Team X wins against Team Y.

	Perfect balance	Imbalance			Strong imbalance		
A	S/M/W	S	M	W	S	M	W
$p_{\leq 0}$	60%	30%	60%	90%	30%	60%	90%
$p_{<0}$	40%	10%	40%	80%	10%	40%	80%
p_{SMF}	41.9%	14.6%	47.4%	52.7%	5.9%	50.0%	34.6%
p^*_{SMF}	11.3%	1.9%	11.6%	14.8%	1.0%	20.0%	8.3%
$p_{\mathrm{SMF}}(16)$	100.0%	91.9%	100.0%	100.0%	62.3%	100.0%	99.9%
$p^*_{\mathrm{SMF}}(16)$	85.2%	25.8%	86.1%	92.3%	15.4%	97.2%	75.1%
$\mathbb{E}[N_{\mathrm{SMF}}]$	6.7	2.3	7.6	8.4	0.9	8.0	5.3
$\mathbb{E}[N^*_{\mathrm{SMF}}]$	1.8	0.3	1.9	2.4	0.2	3.2	1.3

Impact of competitive balance

	Perfect balance			Imbalance			Strong imbalance		
Win prob.	S	M	W	S	M	W	S	M	W
S (Strong)		37.5%	37.5%		50%	80%		70%	90%
M (Middle)	37.5%		37.5%	20%		50%	10%		70%
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	Perfect balance	Imbalance			Strong imbalance		
A	S/M/W	S	M	W	S	M	W
$p_{\leq 0}$	60%	30%	60%	90%	30%	60%	90%
$p_{<0}$	40%	10%	40%	80%	10%	40%	80%
p_{SMF}	41.9%	14.6%	47.4%	52.7%	5.9%	50.0%	34.6%
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$p_{<0}$	40%	10%	40%	80%	10%	40%	80%
p_{SMF}	41.9%	14.6%	47.4%	52.7%	5.9%	50.0%	34.6%
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Impact of forbidding draws

	Perfect balance			Imbalance			Strong imbalance		
Win prob.	S	M	W	S	M	W	S	M	W
S (Strong)		50%	50%		65%	87.5%		80%	94%
M (Middle)	50%		50%	35%		65%	20%		80%
W (Weak)	50%	50%		12.5%	35%		6%	20%	

Table: Win probabilities: p_{XY} is the probability that Team X wins against Team Y. Here, draws are forbidden, so $p_{XY} + p_{YX} = 1$

	Perfect balance	Imbalance			Strong imbalance		
A	S/M/W	S	M	W	S	M	W
$p_{\leq 0}$	60%	30%	60%	90%	30%	60%	90%
$p_{<0}$	40%	10%	40%	80%	10%	40%	80%
p_{SMF}	30.0%	11.6%	32.7%	30.6%	7.1%	40.8%	21.2%
p^*_{SMF}	20.0%	3.9%	21.8%	27.2%	2.4%	27.2%	18.9%
$p_{\mathrm{SMF}}(16)$	99.7%	86.2%	99.8%	99.7%	69.1%	100.0%	97.8%
$p^*_{\mathrm{SMF}}(16)$	97.2%	46.9%	98.0%	99.4%	31.8%	99.4%	96.5%
$\mathbb{E}[N_{\mathrm{SMF}}]$	4.8	1.9	5.2	4.9	1.1	6.5	3.4
$\mathbb{E}[N^*_{\mathrm{SMF}}]$	3.2	0.6	3.5	4.4	0.4	4.4	3.0

Impact of forbidding draws

	Perfect balance			Imbalance			Strong imbalance		
Win prob.	S	M	W	S	M	W	S	M	W
S (Strong)		50%	50%		65%	87.5%		80%	94%
M (Middle)	50%		50%	35%		65%	20%		80%
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$\mathbb{E}[N_{\mathrm{SMF}}]$	4.8	1.9	5.2	4.9	1.1	6.5	3.4
$\mathbb{E}[N^*_{\mathrm{SMF}}]$	3.2	0.6	3.5	4.4	0.4	4.4	3.0

Impact of forbidding draws

	Perfect balance			Imbalance			Strong imbalance		
Win prob.	S	M	W	S	M	W	S	M	W
S (Strong)		50%	50%		65%	87.5%		80%	94%
M (Middle)	50%		50%	35%		65%	20%		80%
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Table: Win probabilities: p_{XY} is the probability that Team X wins against Team Y. Here, draws are forbidden, so $p_{XY} + p_{YX} = 1$

	Perfect balance	Imbalance			Strong imbalance		
A	S/M/W	S	M	W	S	M	W
$p_{\leq 0}$	60%	30%	60%	90%	30%	60%	90%
$p_{<0}$	40%	10%	40%	80%	10%	40%	80%
p_{SMF}	30.0%	11.6%	32.7%	30.6%	7.1%	40.8%	21.2%
p^*_{SMF}	20.0%	3.9%	21.8%	27.2%	2.4%	27.2%	18.9%
$p_{\text{SMF}}(16)$	99.7%	86.2%	99.8%	99.7%	69.1%	100.0%	97.8%
$p^*_{\text{SMF}}(16)$	97.2%	46.9%	98.0%	99.4%	31.8%	99.4%	96.5%
$\mathbb{E}[N_{\text{SMF}}]$	4.8	1.9	5.2	4.9	1.1	6.5	3.4
$\mathbb{E}[N^*_{\text{SMF}}]$	3.2	0.6	3.5	4.4	0.4	4.4	3.0

Impact of the point system: 3-2-1-0

Situation of Team A after Match 2	Probability	SMF	SMF*	SMF^*_{II}
Two wins	$p_{AB}p_{AC}$			
One win and one draw	$p_{AB}d_{AC} + d_{AB}p_{AC}$			
One win and one loss, $\text{GD}_A > 0$	$p_{>0}(p_{AB}p_{CA} + p_{BA}p_{AC})$			
One win and one loss, $\text{GD}_A = 0$	$p_0(p_{AB}p_{CA} + p_{BA}p_{AC})$	✔		
One win and one loss, $\text{GD}_A < 0$	$p_{<0}(p_{AB}p_{CA} + p_{BA}p_{AC})$	✔	✔	
Two draws, two wins on penalties	$\frac{1}{4}d_{AB}d_{AC}$			
Two draws, at least 1 loss on pen.	$\frac{3}{4}d_{AB}d_{AC}$	✔		
One draw & one loss, win on pen.	$\frac{1}{2}(p_{BA}d_{AC} + d_{AB}p_{CA})$	✔		✔
One draw & one loss, loss on pen.	$\frac{1}{2}(p_{BA}d_{AC} + d_{AB}p_{CA})$			
Two losses	$p_{BA}p_{CA}$			

- Suspicion of match fixing **aggravated of type II**: when Team B or C can win the group and eliminate Team A even after drawing its last game and losing on penalties.
- B and C may agree on a draw, say 0-0, and the team leading in the rankings can at no expense **decide** to eliminate Team A by losing the penalty shootout – a situation FIFA surely wants to avoid by all means.

Impact of the point system: 3-2-1-0

Situation of Team A after Match 2	Probability	SMF	SMF*	SMF^*_{II}
Two wins	$p_{AB}p_{AC}$			
One win and one draw	$p_{AB}d_{AC} + d_{AB}p_{AC}$			
One win and one loss, $\text{GD}_A > 0$	$p_{>0}(p_{AB}p_{CA} + p_{BA}p_{AC})$			
One win and one loss, $\text{GD}_A = 0$	$p_0(p_{AB}p_{CA} + p_{BA}p_{AC})$	✔		
One win and one loss, $\text{GD}_A < 0$	$p_{<0}(p_{AB}p_{CA} + p_{BA}p_{AC})$	✔	✔	
Two draws, two wins on penalties	$\frac{1}{4}d_{AB}d_{AC}$			
Two draws, at least 1 loss on pen.	$\frac{3}{4}d_{AB}d_{AC}$	✔		
One draw & one loss, win on pen.	$\frac{1}{2}(p_{BA}d_{AC} + d_{AB}p_{CA})$	✔		✔
One draw & one loss, loss on pen.	$\frac{1}{2}(p_{BA}d_{AC} + d_{AB}p_{CA})$			
Two losses	$p_{BA}p_{CA}$			

- Suspicion of match fixing **aggravated of type II**: when Team B or C can win the group and eliminate Team A even after drawing its last game and losing on penalties.
- B and C may agree on a draw, say 0-0, and the team leading in the rankings can at no expense **decide** to eliminate Team A by losing the penalty shootout – a situation FIFA surely wants to avoid by all means.

Impact of the point system: 3-2-1-0

Proposition

In the 3-2-1-0 point system, the probability of suspicion of match fixing in a given group of 3 is

$$p_{\mathrm{SMF}} := \frac{1}{2}(d_{AB}p_{CA} + p_{BA}d_{AC}) + \frac{3}{4}d_{AB}d_{AC} + p_{\leq 0}\left(p_{AB}p_{CA} + p_{BA}p_{AC}\right);$$

the probability of aggravated suspicion of match fixing in a given group of 3 is

$$p^*_{\mathrm{SMF}} := p_{<0}\left(p_{AB}p_{CA} + p_{BA}p_{AC}\right);$$

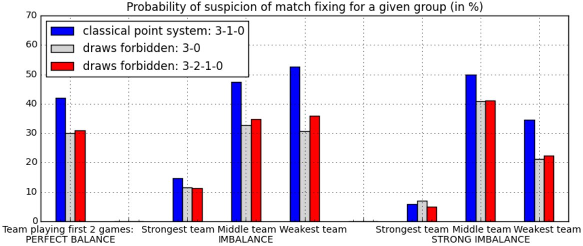
and the probability of aggravated suspicion of match fixing of type II in a given group of 3 is

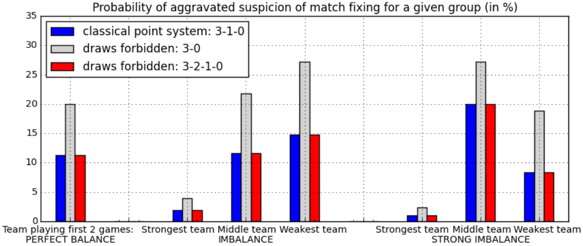
$$p^*_{\mathrm{SMF,II}} := \frac{1}{2}(d_{AB}p_{CA} + p_{BA}d_{AC}).$$

Impact of the point system: 3-2-1-0

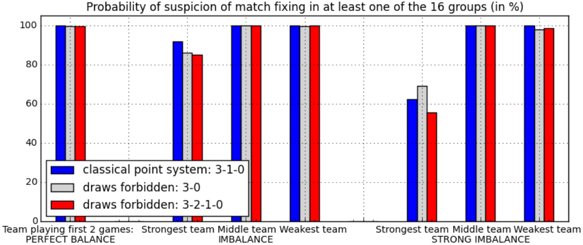
	Perfect balance	Imbalance			Strong imbalance		
A	S/M/W	S	M	W	S	M	W
$p_{\leq 0}$	60%	30%	60%	90%	30%	60%	90%
$p_{<0}$	40%	10%	40%	80%	10%	40%	80%
p_{SMF}	30.9%	11.2%	34.7%	35.8%	4.9%	41.0%	22.4%
p^*_{SMF}	11.3%	1.9%	11.6%	14.8%	1.0%	20.0%	8.3%
$p^*_{\mathrm{SMF,II}}$	9.4%	9.3%	10.5%	15.8%	0.6%	8.0%	11.8%
$p_{\mathrm{SMF}}(16)$	99.7%	85.0%	99.9%	99.9%	55.4%	99.98%	98.7%
$p^*_{\mathrm{SMF}}(16)$	85.2%	25.8%	86.1%	92.3%	15.4%	97.2%	75.1%
$p^*_{\mathrm{SMF,II}}(16)$	79.4%	78.8%	83.0%	93.6%	9.2%	73.7%	86.6%
$\mathbb{E}[N_{\mathrm{SMF}}]$	5.0	1.8	5.6	5.7	0.8	6.6	3.6
$\mathbb{E}[N^*_{\mathrm{SMF}}]$	1.8	0.3	1.9	2.4	0.2	3.2	1.3
$\mathbb{E}[N^*_{\mathrm{SMF,II}}]$	1.5	1.5	1.7	2.5	0.1	1.3	1.9

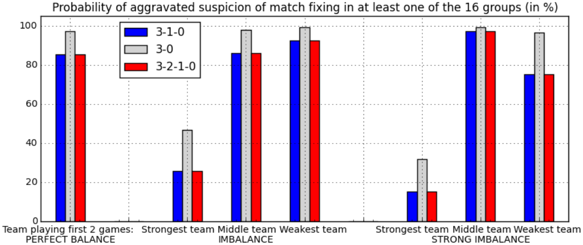
Summary


Probability of suspicion of match fixing for a given group (in %)
classical point system: 3-1-0
draws forbidden: 3-0
draws forbidden: 3-2-1-0
70
60
50
40
30
20
10
0
Team playing first 2 games:
PERFECT BALANCE
Strongest team
Middle team
Weakest team
IMBALANCE
Strongest team
Middle team
Weakest team
STRONG IMBALANCE

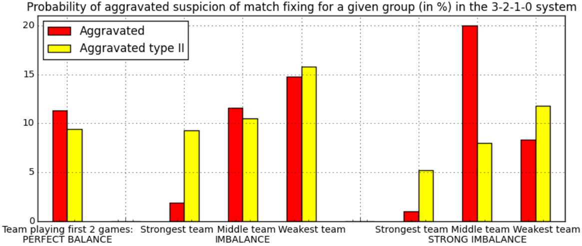

Probability of aggravated suspicion of match fixing for a given group (in %)
classical point system: 3-1-0
draws forbidden: 3-0
draws forbidden: 3-2-1-0
35
30
25
20
15
10
5
0
Team playing first 2 games:
PERFECT BALANCE
Strongest team
Middle team
Weakest team
IMBALANCE
Strongest team
Middle team
Weakest team
STRONG IMBALANCE

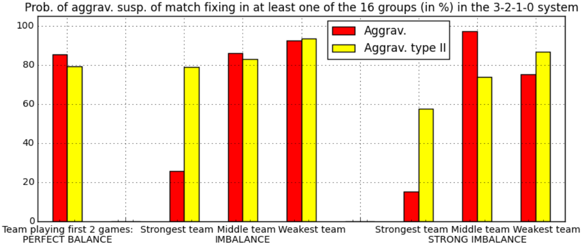
Summary


Probability of suspicion of match fixing in at least one of the 16 groups (in %)
100
80
60
40
20
0
classical point system: 3-1-0
draws forbidden: 3-0
draws forbidden: 3-2-1-0
Team playing first 2 games:
PERFECT BALANCE
Strongest team
Middle team
Weakest team
IMBALANCE
Strongest team
Middle team
Weakest team
STRONG IMBALANCE

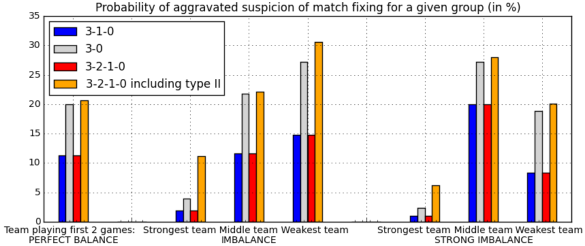

Probability of aggravated suspicion of match fixing in at least one of the 16 groups (in %)
100
80
60
40
20
0
3-1-0
3-0
3-2-1-0
Team playing first 2 games:
PERFECT BALANCE
Strongest team
Middle team
Weakest team
IMBALANCE
Strongest team
Middle team
Weakest team
STRONG IMBALANCE

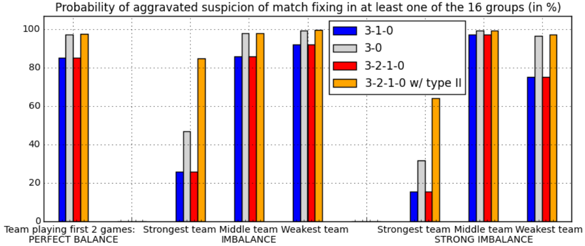
Summary


Probability of aggravated suspicion of match fixing for a given group (in %) in the 3-2-1-0 system
Aggravated
Aggravated type II
0
5
10
15
20
Team playing first 2 games:
PERFECT BALANCE
Strongest team
Middle team
Weakest team
IMBALANCE
Strongest team
Middle team
Weakest team
STRONG IMBALANCE


Prob. of aggrav. susp. of match fixing in at least one of the 16 groups (in %) in the 3-2-1-0 system
Aggrav.
Aggrav. type II
0
20
40
60
80
100
Team playing first 2 games:
PERFECT BALANCE
Strongest team
Middle team
Weakest team
IMBALANCE
Strongest team
Middle team
Weakest team
STRONG IMBALANCE

Summary


Probability of aggravated suspicion of match fixing for a given group (in %)
3-1-0
3-0
3-2-1-0
3-2-1-0 including type II
0
5
10
15
20
25
30
35
Team playing first 2 games:
PERFECT BALANCE
Strongest team
Middle team
Weakest team
IMBALANCE
Strongest team
Middle team
Weakest team
STRONG IMBALANCE


Probability of aggravated suspicion of match fixing in at least one of the 16 groups (in %)
3-1-0
3-0
3-2-1-0
3-2-1-0 w/ type II
0
20
40
60
80
100
Team playing first 2 games:
PERFECT BALANCE
Strongest team
Middle team
Weakest team
IMBALANCE
Strongest team
Middle team
Weakest team
STRONG IMBALANCE

Summary

- **The most important factor impacting suspicion of match fixing is the schedule**: the probability of SMF is minimized when it is the *a priori* strongest team that plays the first 2 games.
- **Forbidding draws (3-0 point system) decreases the probability of SMF, but increases the probability of aggravated SMF**. Note however that in the case of strong imbalance and Team A being the *a priori* strongest team, forbidding draws actually increases the probability of SMF.
- Surprisingly, **compared to the 3-0 point system, the probability of SMF is usually slightly larger in the 3-2-1-0 point system**, except when Team A is the *a priori* strongest team.

Alternate formats

- Assuming a 48 team World Cup, what alternate formats could FIFA use which would significantly decrease, or even eradicate, the risk of collusion?
- All the formats listed below take into account the requirement that the tournament should not last too long; the total number of matches should not exceed, say, 100.
- This precludes the classical round-robin format with 8 groups of 6 – in this format the group stage only would feature 120 games! However, see Format 6.

Format 1: 12 groups of 4

- The 12 group winners, 12 runners-up, and 8 best third-place teams would advance to the round of 32.
- 72 group matches, total of 104 games, the World Cup would then last at least one more week, assuming four group matches per day.
- Both the current format (1998-2018, 64 games) and the 3-team group format (80 games) can be completed in 32 days.
- Since the number of groups would not be a power of 2, the KO bracket would be unbalanced:
  - Some group winners would play against third-placed teams in the round of 32, while other group winners would play against runners-up.
  - Some runners-up would play against each other.

  (like in 1986, 1990, and 1994 FIFA World Cups, and 2016 and 2020 UEFA Euros).
- See Guyon (2018) for a detailed study of such unbalanced brackets.

Format 2: 12 groups of four, only 16 teams advance

- The 12 group winners and 4 best runners-up would advance to the round of 16.
- Decreases the total number of matches to 88 from 104.
- Only a third of the teams would advance to the KO stage.
- The KO bracket would still be unbalanced.

Format 3: 16 groups of 3, only group winners advance

- This would eliminate the risk of collusion.
- The last group match may be a dead rubber (when Team A has two wins).
- There is still a fairness issue: When Team A has one win and one draw (say, against B and C, respectively), Team B is already eliminated before the last group match and may not give its best effort to defeat C. If C wins the group after beating an unmotivated Team B, Team A may feel aggrieved.
- In this format a team (B in the above example) can be eliminated after playing just one game.
- Finally, the WC winner would play only 6 matches, one less than in the current format (1998–2018).

Remark

A classical procedure to avoid that the last group game be a dead rubber consists of enforcing a flexible schedule where the team that loses the first game, if any, plays the 2nd group game. However, does not help decrease risk of collusion when 2 teams advance to next phase.

Format 4: 16 groups of 3, all teams advance

- Group winners would get a bye and directly advance to the round of 32, while runners-up and third-placed teams would advance to a playoff round, whose winner would qualify for the round of 32.
- Total 96 matches, which could fit in 38 days.
- Would indeed eliminate the risk of collusion.
- All teams would play a minimum of 3 games.
- The winner of the tournament would need to play 7 or 8 matches.
- Coaches, fans, TV networks and FIFA might not like the idea of group winners getting a bye, though. Those best teams would not be seen on TV in the playoff round.
- If the group winner is Team A, they'd have at least 12 rest days between their last group match and their first knockout game.

Format 5: 16 groups of 3, seed KO bracket based on performance across groups.

- Guyon (2018) investigates the idea of seeding the KO stage based on performance across groups and illustrates it for 24-team UEFA Euros.
- Group winners and runners-up would be ranked based on group stage performance (say for instance: points, goal differential, and number of goals scored, in that order).
- Then best group winner would play against lowest-ranked runner-up, second best group winner against second lowest-ranked runner-up, etc.
- Incentivizes all teams to perform at their best during the group stage, even if they know that a draw or even a 0-1 loss would be enough to advance to the KO stage, thus significantly decreasing the risk of collusion.
- Logistics problem: Teams and fans could not plan ahead when and where they will be playing during the KO stage if they advance.
- Teams playing the last match of all groups may be unfairly advantaged.
- More soccer-free days would be needed between the group stage and the round of 32.

Format 6: 8 groups of 6, but each team plays only 3 teams in their group

- Each group of 6 is divided into 2 balanced subgroups of 3 teams.
- All teams in Subgroup 1 play against all teams in Subgroup 2.
- The best 2 teams over all advance to the KO stage (they could be both from the same subgroup).
- To enforce balance, 6 pots of 8 teams could be formed based on the new FIFA rankings, from Pot 1 (the 8 highest-ranked teams) to Pot 6 (the 8 lowest-ranked teams). One subgroup would contain teams from Pots 1, 4 and 5 (or 6), while the other subgroup would contain teams from Pots 2, 3 and 5 (or 6).
- Each team plays 3 group matches.
- All teams play simultaneously on Match Day 3 to prevent collusion
- Predetermined bracket routes are posssible.
- By splitting groups, number of group matches would decrease to 72 from 120. Total of 88 games, which could fit in about 35 days.

Conclusion

- We have quantified the risk of collusion in a group of 3 teams playing a single round-robin tournament, where 2 teams advance to next phase.
- The best way to minimze the risk of collusion is to enforce that **the team that plays the first 2 group matches is the a priori strongest team in the group, especially if the group is strongly imbalanced**.
- However this may be deemed unfair to that team as it would be the only one vulnerable to collusion. This would also mean that Match Day 3 of the World Cup would feature none of the seeded teams.
- We have quantified the impact of competitive imbalance.
- We have also quantified by how much the risk of collusion would decrease if FIFA does not use the traditional 3-1-0 point system but adopts alternate point systems that forbid draws, the 3-0 and 3-2-1-0 point systems.
- Even if it looks appealing on paper, **the 3-2-1-0 point system does not in general do a better job at decreasing the risk of collusion than the 3-0 point system**.
- Finally, we have described practical **alternate formats** for a 48 team World Cup that would eliminate or strongly decrease the risk of collusion, with groups of 3, 4, or 6 teams.

- If FIFA wants to keep groups of 3 with the best 2 teams advancing, Format 5 seems the best solution to minimize the risk of collusion, where the **KO bracket is seeded based on performance across groups**.
- 16 groups $\Longrightarrow$ risk of collusion in at least one group is very high, even in the most favorable case where all groups are strongly imbalanced and in every group Team A is the *a priori* strongest team in the group ($\approx 60\%$).
- This proves, by the numbers, that the introduction of groups of 3 is a terrible step back in the history of the World Cup. **Not only it makes the "disgrace of Gijón" possible again, but it makes the risk of its repetition very high**.
- Of course, not all teams would collude if given the opportunity, but even suspicion of match fixing may seriously tarnish the World Cup, as unpredictability of the outcome is fundamental to its popularity, and to sport's popularity in general.
- It is FIFA's responsibility to build a fair World Cup. **It is not too late for FIFA to review the format of the 2026 World Cup**.

New York Times, June 11, 2018

The New York Times

TheUpshot

Why Groups of 3 Will Ruin the World Cup (So Enjoy This One)

Unless FIFA changes course, the risk of collusion will be much higher in future Cups.

A new format with 48 teams, starting in 2026 but possibly as soon as the next World Cup, raises a serious fairness issue in the chase for the World Cup trophy. Alexander Zemlianichenko/Associated Press

By Julien Guyon

June 11, 2018

68

If you're a World Cup aficionado, you may want to take the time to savor this one, because it may be the last version with a format — groups of four teams — that has a fair and consistently exciting

New York Times, June 26, 2018

The New York Times

TheUpshot

FIFA, We Fixed Your World Cup Collusion Problem for You

Readers tackle a math puzzle on how to devise a fair tournament with 48 teams.

Can FIFA find its way to a fair tournament format before the next World Cup? Spain goalkeeper David De Gea looked lost after giving up a goal June 15 against Portugal. Thanassis Stavrakis/Associated Press

By Toni Monkovic

June 26, 2018

38

The World Cup hardly seems broken, and yet FIFA seems desperate to fix it.

FIFA, world soccer's governing body, has decided that the World

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