Integrated Break and Carryover Minimization Problem in Round Robin Tournaments

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Round Robin Tournaments (RRTs)

- Every team plays with every other team (for once: Single RRT / twice: Double RRT)
- We are only interested in time-constrained single RRTs
- If number of teams is n
  - In each round, every team plays one game ($n/2$ games per round)
  - In a double RRT, the tournament lasts for $2 \cdot (n-1)$ rounds.
  - In a double RRT, $n \cdot (n-1)$ games are played in total. (For exp: 306 games if $n = 18$)
- Two popular fairness criteria in time-constrained RRTs.
  - Number of breaks
  - Carryover effects (coe) value

Number of Breaks

	Rounds										
	R1	R2	R3	R4	R5	R6	R7	R8	R9	R10	R11
Games	D-C	K-C	B-D	I-L	K-H	H-I	D-J	G-J	F-H	D-I	C-E
	K-L	I-A	H-J	J-B	I-J	B-C	K-G	L-C	K-B	F-K	I-K
	F-J	B-F	F-L	G-A	A-C	E-F	A-F	B-A	C-G	G-L	J-A
	E-B	L-D	C-I	C-F	L-B	L-A	I-B	D-K	L-J	B-H	G-B
	I-G	G-H	A-K	H-D	F-G	J-K	E-L	F-I	A-D	E-A	D-F
	A-H	J-E	E-G	K-E	D-E	G-D	C-H	H-E	E-I	J-C	H-L

Break: If a team plays two home or away matches in two successive rounds, the alternating H/A pattern for the team is said to be *broken* and the team has a "break".

		Rounds											
		R1	R2	R3	R4	R5	R6	R7	R8	R9	R10	R11	# Break
Teams	A	H	A	H	A	H	A	H	A	H	A	**A**	1
	B	A	H	**H**	A	**A**	H	A	H	A	H	A	2
	C	A	**A**	H	**H**	A	**A**	H	A	H	A	H	3
	D	H	A	**A**	**A**	H	A	H	**H**	A	H	**H**	4
	E	H	A	H	A	**A**	H	**H**	A	H	**H**	A	3
	F	H	A	H	A	H	A	**A**	H	**H**	**H**	A	3
	G	A	H	A	H	A	H	A	H	A	H	**H**	1
	H	A	**A**	H	**H**	A	H	A	H	A	**A**	H	3
	I	H	**H**	A	H	**H**	A	H	A	**A**	**A**	H	4
	J	A	H	A	H	A	H	A	**A**	**A**	H	**H**	3
	K	H	**H**	A	H	**H**	A	H	A	H	A	**A**	3
	L	A	H	A	**A**	H	**H**	A	H	**H**	A	**A**	4

$b_{i,r}$ (binary): occurrence of a break for team i in round r

Number of breaks: $\sum_i \sum_r b_{i,r}$

# of breaks: 34

Break Minimization

- [1a] de Werra (1981)
  - Canonical schedules minimize the number of breaks.
- [1b] Elf et al. (2003)
  - *A new break minimization problem*: Given a feasible tournament schedule without home-away assignment, they find H/A pattern that minimizes the number of breaks.
  - They conjecture that this problem is NP-hard.
- [1c] Miyashiro and Matsui (2005)
  - They propose a polynomial-time algorithm which finds a home-away assignment for a given feasible tournament schedule if a solution with $n-2$ breaks exists for the schedule, else returns “infeasible”.

Carryover Effects (coe) Value

		Rounds										
		R1	R2	R3	R4	R5	R6	R7	R8	R9	R10	R11
Teams	A	H	I	K	G	C	L	F	B	D	E	J
	B	E	F	D	J	L	C	I	A	K	H	G
	C	D	K	I	F	A	B	H	L	G	J	E
	D	C	L	B	H	E	G	J	K	A	I	F
	E	B	J	G	K	D	F	L	H	I	A	C
	F	J	B	L	C	G	E	A	I	H	K	D
	G	I	H	E	A	F	D	K	J	C	L	B
	H	A	G	J	D	K	I	C	E	F	B	L
	I	G	A	C	L	J	H	B	F	E	D	K
	J	F	E	H	B	I	K	D	G	L	C	A
	K	L	C	A	E	H	J	G	D	B	F	I
	L	K	D	F	I	B	A	E	C	J	G	H

c_{ij}: number of carrovers carryovers Team j receives from Team i

coe value : $\sum_{ij} c_{ij}^2$

Carryover effect: Assume Team A plays against Team B in the previous round, and against Team C in this round. This means Team C receives a carryover effect from Team B in the current round.

		Teams												
		A	B	C	D	E	F	G	H	I	J	K	L	$\Sigma_j c_{ij}^2$
Teams	A	0	1	2	0	2	3	1	0	2	0	1	0	24
	B	1	0	0	1	0	2	0	2	3	1	0	2	24
	C	2	2	0	0	1	0	1	0	1	1	0	4	28
	D	0	1	0	0	1	2	1	0	0	3	4	0	32
	E	2	0	1	3	0	2	1	2	0	1	0	0	24
	F	1	2	2	2	2	0	0	0	2	0	0	1	22
	G	1	0	1	1	3	0	0	1	0	3	1	1	24
	H	0	2	0	0	2	0	1	0	2	1	3	1	24
	I	2	1	1	0	0	2	0	2	0	0	2	2	22
	J	0	1	1	1	1	0	3	3	0	0	1	1	24
	K	1	0	0	4	0	0	3	1	2	1	0	0	32
	L	2	2	4	0	0	1	1	1	0	1	0	0	28

coe: 308

Carryover Effects (coe) Value

- Carry-over effects value optimization
  - [2a] Russell (1980)
  - [2b] Anderson (1997)
  - [2c] Guedes ve Ribeiro (2011)
  - [2d] Lambrechts et al. (2016)
- Carryover effect and practical implications
  - [3a] Goossens (2018)
- Impact of carryover effect on match results
  - [4a] Goossens ve Spieksma (2012)

n	*n (n-1)*	best coe
8	56	56 [2a]
10	90	108 [2b]
12	132	160 [2c]
14	182	234 [2b]
16	240	240 [2a]
18	306	340 [2b]
20	380	380 [2b]

Canonical schedules maximize coe value.
E.g. $n = 18$ → coe $= 3876$

Integrated Break and Carryover Minimization

- **Problem:** Determine a <u>*schedule*</u> to minimize break and carryover simultaneously
- <u>*Schedule*</u> implies: (1) the games of each round and (2) H/A pattern of each team
- [5a] Günneç and Demir (2018)

n	any # of break	# of break for each team ≤ 1
8	56 [2a]	104 [5a]
10	108 [2b]	192 [5a]
12	160 [2c]	318 [5a]
14	234 [2b]	446 [5a]
16	240 [2a]	626 [5a]
18	340 [2b]	944 [5a]
20	380 [2b]	—

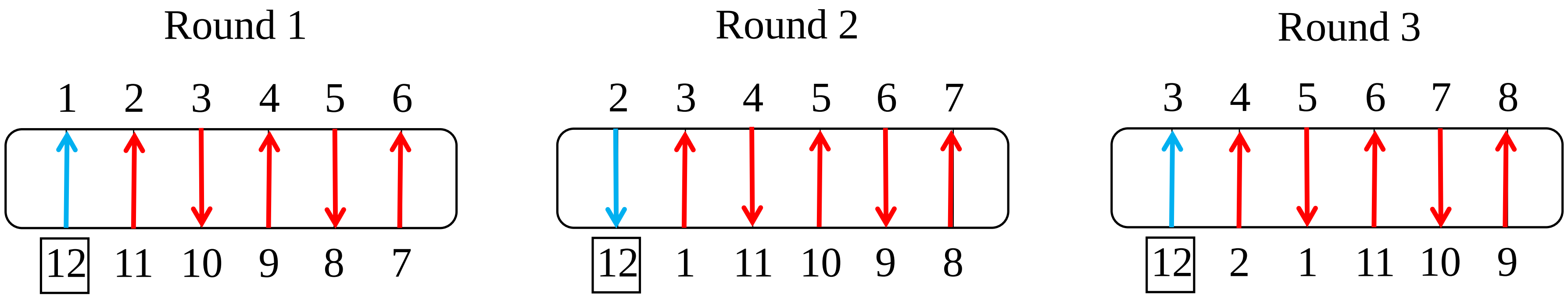
Solution Method

How to solve:

1. Construct a canonical schedule (with *circle method*)
2. Run a *round swapping* algorithm which randomizes the order of rounds to construct schedules isomorphic to the canonical schedule
3. Identify the schedule with the smallest coe value
4. Solve a *break minimization problem* for this particular schedule

Solution Method – STEP 1: Circle Method

- Rotate in counter-clockwise direction



Round	Games					
1	12-1	11-2	3-10	9-4	5-8	7-6
2	2-12	1-3	4-11	10-5	6-9	8-7
3	12-3	2-4	5-1	11-6	7-10	9-8

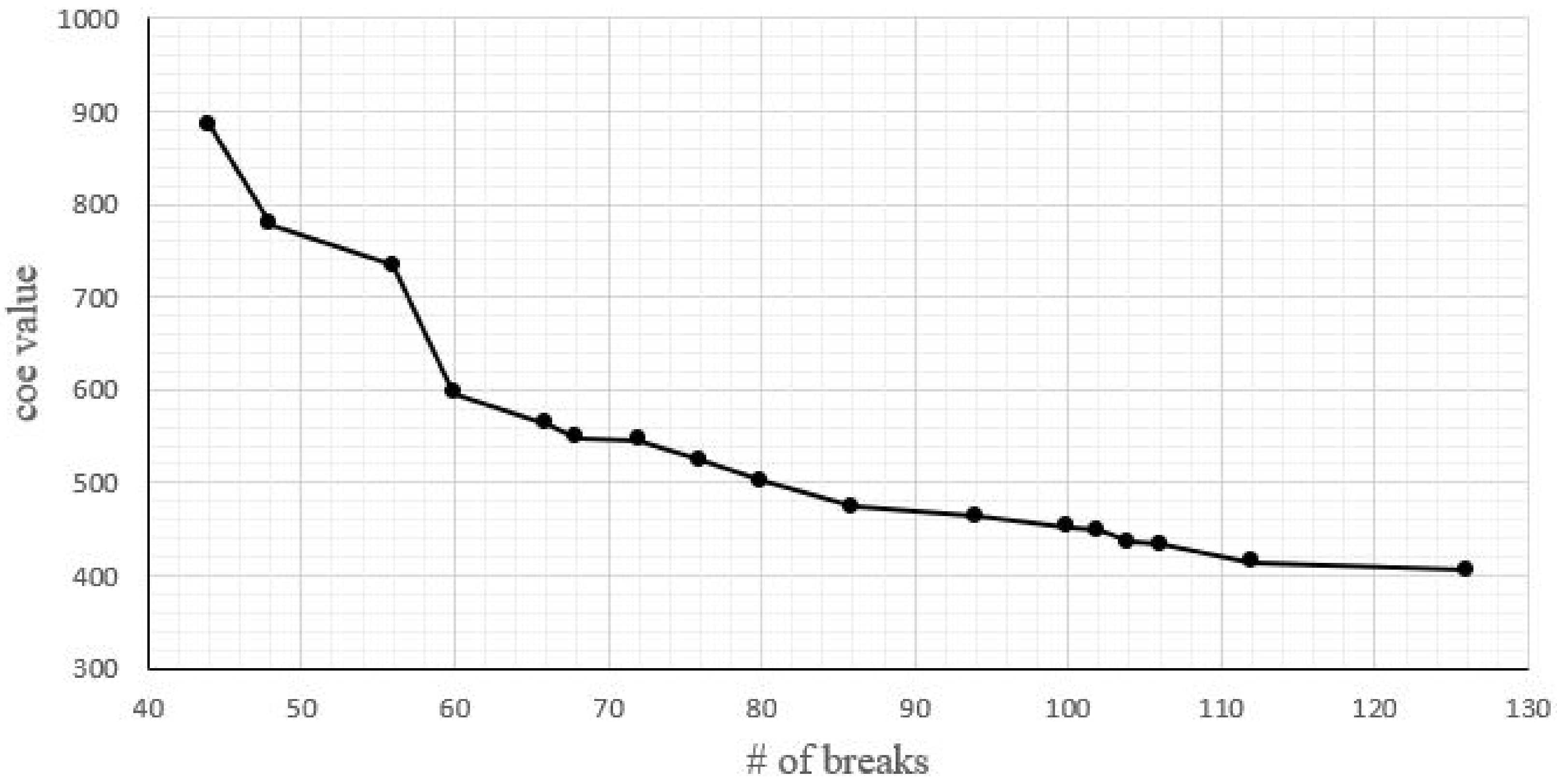
Solution Method – STEP 2: Round Swapping

- Randomize rounds

Round 1	12 vs 1	11 vs 2	3 vs 10	9 vs 4	5 vs 8	7 vs 6
Round 2	2 vs 12	1 vs 3	4 vs 11	10 vs 5	6 vs 9	8 vs 7
Round 3	12 vs 3	2 vs 4	5 vs 1	11 vs 6	7 vs 10	9 vs 8
Round 4	4 vs 12	3 vs 5	6 vs 2	1 vs 7	8 vs 11	10 vs 9
Round 5	12 vs 5	4 vs 6	7 vs 3	2 vs 8	9 vs 1	11 vs 10
Round 6	6 vs 12	5 vs 7	8 vs 4	3 vs 9	10 vs 2	1 vs 11
Round 7	12 vs 7	6 vs 8	9 vs 5	4 vs 10	11 vs 3	2 vs 1
Round 8	8 vs 12	7 vs 9	10 vs 6	5 vs 11	1 vs 4	3 vs 2
Round 9	12 vs 9	8 vs 10	11 vs 7	6 vs 1	2 vs 5	4 vs 3
Round 10	10 vs 12	9 vs 11	1 vs 8	7 vs 2	3 vs 6	5 vs 4
Round 11	12 vs 11	10 vs 1	2 vs 9	8 vs 3	4 vs 7	6 vs 5

Solution Method – STEP 3: Identifying Best coe

# of teams (n)	10		12		14		16		18	
	Coe	Break	Coe	Break	Coe	Break	Coe	Break	Coe	Break
Best coe	**136**	26	**192**	60	**254**	68	**330**	118	**406**	126
Best break	468	**8**	392	**16**	634	**22**	660	**40**	884	**44**



Solution Method – STEP 4: Break Min. Problem

- Minimize the sum of breaks (for a given schedule)

Subject to

- If one of the two opposing teams plays at home (away), the other team plays away (at home).
- If a team plays at home (away) in two successive rounds, a break occurs.
- [Optional] Number of breaks for each team should not exceed a certain value.

Solution Method – STEP 4: Break Min. Problem

- *n* : Number of teams (even)
- Sets
  - *T* : Teams, *i* = 1, …, *n*   *R* : Rounds, *r* = 1, …, *n*-1
- Parameters
  - $x_{i,j,r} = 1$ if team i plays against team j in round r, and 0 otherwise.
- Decision variables
  - $h_{i,r} =$ 1 if team i plays at home in round r, and 0 otherwise
  - $b_{i,r} =$ 1 if there is an occurrence of break for team i in round r, and 0 otherwise
  - z : sum of breaks

Solution Method – STEP 4: Break Min. Problem

$$\min \quad z = \sum_{i \in T} \sum_{r \in R} b_{i,r} \tag{1}$$

Subject to:

$$h_{i,r} + h_{j,r} \geq x_{i,j,r} \quad \forall i, j \in T \mid i \neq j, \forall r \in R \tag{2}$$

$$2 - h_{i,r} + h_{j,r} \geq x_{i,j,r} \quad \forall i, j \in T \mid i \neq j, \forall r \in R \tag{3}$$

$$h_{i,r} + h_{i,r+1} \leq 1 + b_{i,r+1} \quad \forall i \in T, \forall r \in R \tag{4}$$

$$2 - h_{i,r} - h_{i,r+1} \leq 1 + b_{i,r+1} \quad \forall i \in T, \forall r \in R \tag{5}$$

$$h_{i,r}, b_{i,r} \in \{0, 1\} \quad \forall i \in T, \forall r \in R \tag{6}$$

Computational Experiments

- Intel i5-4570 CPU 3.2 GHz
- 8Gb RAM
- GAMS/Gurobi

- For each $n = 10, 12\ 14, 16, 18$
  - We randomized the rounds *1 million times* and computed coe value for each.
  - We solved the break minimization problem for the schedule with the best coe value.
  - We compared our results with “[5a] Günneç and Demir (2018)”.

Computational Experiments

# of teams (n)	10	12	14	16	18
$n \cdot (n-1)$	90	132	182	240	306
any # of breaks	108 [2b]	160 [2c]	234 [2b]	240 [2a]	340 [2b]
$b_i \leq 1$	192 (8) [5a]	318 (10) [5a]	446 (12) [5a]	626 (14) [5a]	944 (16) [5a]
$b_i \leq 2$	144 (12) [5a]	212 (18) [5a]	344 (26) [5a]	472 (30) [5a]	646 (30) [5a]
$b_i \leq 3$	144 (12) [5a]	212 (16) [5a]	302 (24) [5a]	396 (34) [5a]	556 (40) [5a]

Best coe values in the literature (values in parenthesis are the number of breaks)

# of teams (n)	6% 10	0%	9% 12	13%	16% 14	8%	17% 16	0%	27% 18	15%
	Coe	Break	Coe	Break	Coe	Break	Coe	Break	Coe	Break
$b_i \leq 2$		12		18		infsbl		infsbl		infsbl
$b_i \leq 3$	136	12	192	18	254	26	330	34	406	46
b_i *unbounded*		12		18		26		32		42

Our results

Conclusion: A template for $n = 18$

Round	Games								
1	11-1	10-12	13-9	8-14	15-7	6-16	17-5	4-18	2-3
2	1-10	9-11	12-8	7-13	14-6	5-15	16-4	3-17	18-2
3	6-1	**7**-5	4-**8**	**9**-3	10-**2**	11-18	17-12	13-16	15-14
4	1-9	8-10	**7**-11	12-6	5-13	14-4	3-15	16-**2**	18-17
5	**1**-7	6-8	9-5	4-**10**	11-3	2-12	13-18	17-14	15-16
6	12-1	**13**-11	10-**14**	**9**-15	16-**8**	7-17	18-6	5-2	3-4
7	1-13	14-12	11-**15**	**10**-16	17-9	8-18	2-7	6-3	4-5
8	**1**-17	18-**16**	15-2	3-14	13-4	12-**5**	**11**-6	7-10	9-8
9	16-1	17-15	14-18	2-13	**3**-12	4-11	5-**10**	6-9	8-7
10	1-5	**6**-4	7-3	**8**-2	9-**18**	10-17	11-16	12-**15**	13-14
11	14-1	15-13	16-12	17-11	18-10	2-9	3-8	4-7	5-6
12	1-4	**5**-3	6-2	7-18	8-17	9-16	**15**-**10**	11-14	12-**13**
13	18-1	17-**2**	3-**16**	4-15	14-5	13-6	**7**-12	**11**-8	10-9
14	1-3	2-4	5-18	6-17	16-7	15-**8**	9-14	**10**-13	12-11
15	**1**-15	14-16	13-**17**	18-12	11-2	3-10	4-9	8-5	7-6
16	**8**-1	9-7	10-**6**	5-11	12-4	**3**-13	2-14	15-18	17-**16**
17	1-2	18-3	4-17	16-5	6-15	7-**14**	13-8	**9**-12	11-10

coe value: 406
number of breaks: 42
(2 teams with no break,
2 teams with one break,
6 teams with two breaks,
6 teams with three breaks,
2 teams with five breaks).

Teams having break are highlighted with **bold text**.

Thank You…

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