Three Point Shooting and Efficient Mixed Strategies: An Indirect Evolutionary Game Theoretic Approach

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Presentation: MathSport International 2019 Conference

Athens, Greece July 3, 2019

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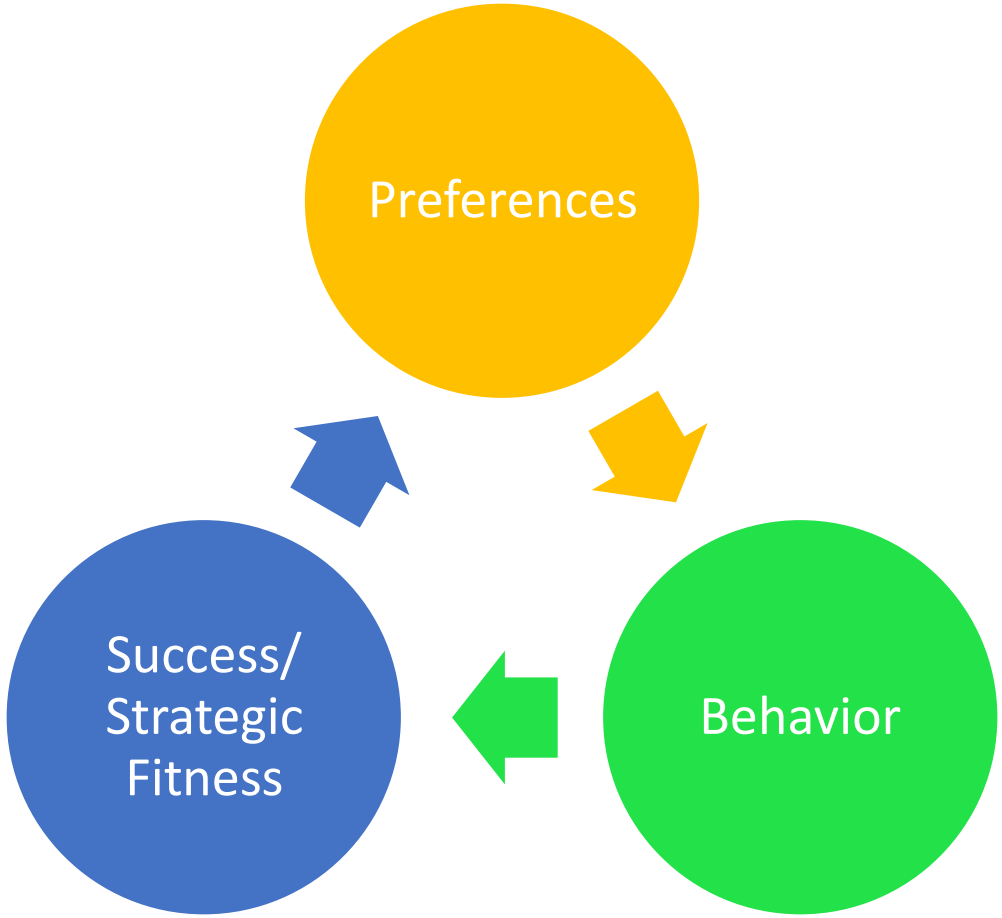
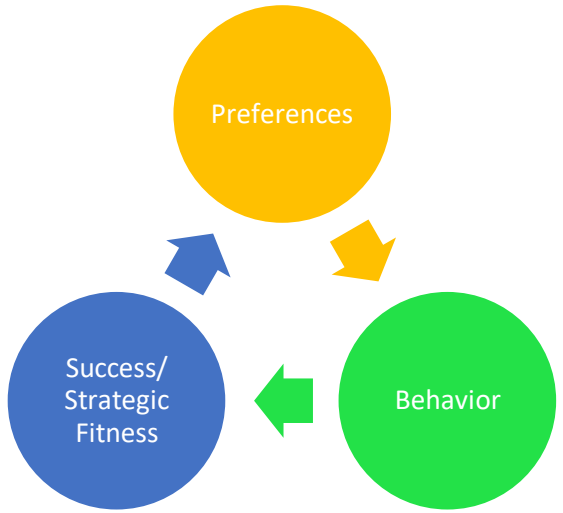
Evolutionary Game Theory and Basketball

- Preferences are a function of strategic fitness (e.g., Guth and Yaari (1992), Bisin and Verdier (2001))
  - Survival of a coach depends upon their ability to win
- "Evolutionary game theory has produced an impressive body of abstract results. Its continued relevance now depends upon the ability to use these results in more concrete economic settings."
  - Samuelson (2003):
- In this paper we examine applicability of this theory to NBA basketball



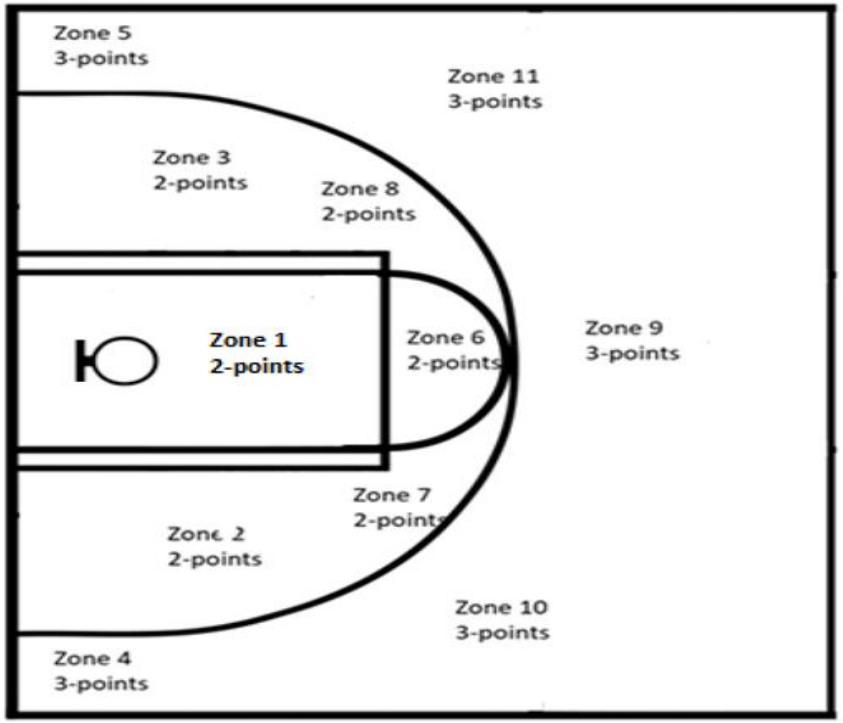
Preview of Results

- Estimate implied preferences assuming strategic fitness
  - Estimate implied coefficient of risk aversion (CRRA) from closest Stackelberg Equilibrium to observed strategies
  - Risk aversion => employ strategies that trade off expected point production to reduce risk
- Main Result: Successful defenses induce higher levels of risk aversion from the opposing team's offence which in turn allows the team to reduce their own offensive risk aversion
  - Unsupervised Learning Insights: Identifies strategies associated with successful defenses (i.e., high +ve CRRA versus high –ve CRRA factor loadings)
- Equilibrium provides statistically significant (< 0.0001) out of sample predictions (post season)

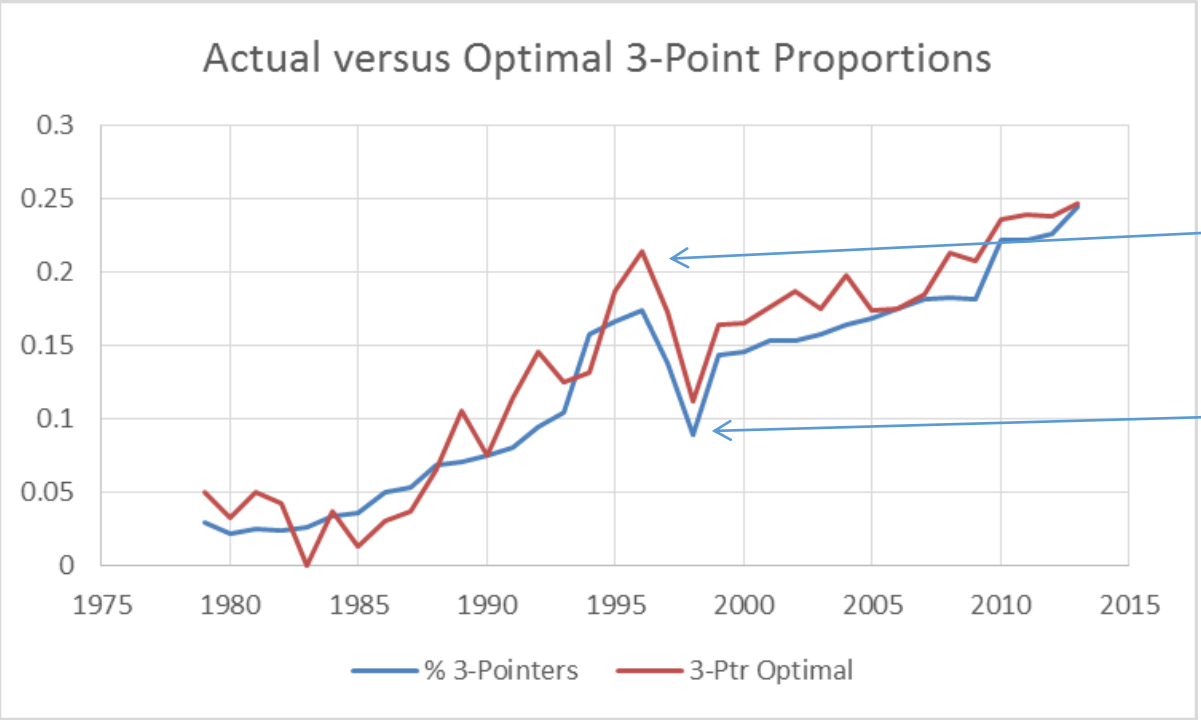


Basketball as a Nash Equilibrium with Commitment

- We model basketball as a Stackelberg equilibrium *with commitment*
  - Conitzer and Sandholm (2006), Conitzer (2016))
- Strategy is a vector, $\omega = (\omega_1, \ldots\ldots \omega_{11})$, of relative proportions of shots taken from each zone
  - *Restrict to the class of the CRRA preferences to identify efficient strategies prior to start of play*
- Equilibrium is a *pair* of mixed strategies
  - Component 1: Team A's offensive strategy when playing against team B's defense
  - Component 2: Team B's offensive strategy when playing against team A's defense.
  - Equilibrium strategies can be identified relative to exogenous preferences (e.g., Fichman and O'Brien(2017, 2018))
- Current paper we estimate CRRA preference coefficient from *the equilibrium that best describes observed strategies*



Relevance of Mean/Variance Statistics for Basketball



Source: "Investing in Three Point Shooting: A Strategic Portfolio Management Approach" Mark Fichman and John O'Brien Journal of Sports Analytics, Vol 4, No 2, 2018

Mean/Variance/Covariance world provides a nice description/prediction of the evolution of the 3-point shot within the NBA

1994-95

• Shortened the three-point line (22 feet in the corners extending to 23 feet, nine inches at the top of the key) to a uniform 22 feet around the basket.

1997-98

• The three-point line, 22 feet from the basket, lengthened to its original distance of 23 feet, nine inches, except in the corners, where the distance remained 22 feet.

Preferences were exogenously fixed by imposing the Sharpe Ratio (SR) on the mixed strategy optimization problem (E(Payoff)/Volatility = SR)

Solving for Equilibrium Strategies using Mean Variance Statistics

- Preferences are restricted to the class of Iso-elastic utility functions
  - Constant Relative Risk Aversion (CRRA)
- $U_\lambda(x) = \frac{x^{1-\lambda}}{1-\lambda}, \quad \lambda > 0 \text{ and } \lambda \neq 1$
- $U_\lambda(x) = \ln(x), \quad \lambda = 1$
- **Definition:** Equivalent utility functions for CRRA are approximated via a second-order Taylor series as follows (e.g., Choi 2006).
- $U(\mu_0, \sigma_0) = u(\mu_0) + 0.5u''(\mu_0)\sigma_0{}^2$

- **Definition:** A strategy is efficient if the vector of weights ω maximize the expected equivalent utility of points made net of points given up for some CRRA coefficient λ.

$$Maximize\ w.r.t.\ \omega\ \ U_\lambda(\mu_d, \sigma_d)$$

Subject to: $\sum_j \omega_j = 1$

Where

$\omega_j \geq 0$

Results: Equilibrium Example

	CRRA Offense	CRRA Defense	Wins (W)	Losses (L)	W/L %	Ortg	Drtg
CRRA Equilibrium							
Eastern Conference							
Toronto (TOR)	1.3	1.84	51	31	0.622	113.2	108.68
Brooklyn Nets (BKN)	0.86	0.51	20	62	0.244	104.61	111.34
Western Conference							
Golden State Warriors (GSW)	0.98	1.81	67	15	0.817	116.26	104.57
PhoenixSuns (PHX)	0.73	0.94	24	58	0.293	107.32	112.79
Offensive rating (ORtg) = Estimated points scored per 100 possessions							
Defensive rating (DRtg) = Estimated points allowed per 100 possessions							

Note: 1 is the log optimal solution (2016/17 Season)

Team		Offensive CRRA (Mean)	Offensive CRRA (Std Deviation)	Defensive CRRA (Mean)	Defensive CRRA (Std Deviation)
Atlanta Hawks	ATL	1.08	1.36	1.17	1.35
Brooklyn Nets	BKN	0.86	1.08	0.51	0.76
Boston Celtics	BOS	1.38	1.23	1.13	0.65
Charlotte Hornets	CHA	0.99	1.00	0.87	1.03
Chicago Bulls	CHI	0.90	1.05	1.39	1.09
Cleveland Cavaliers	CLE	1.28	1.28	0.90	1.05
Dallas Mavericks	DAL	1.60	1.43	0.94	1.10
Denver Nuggets	DEN	1.44	1.06	1.35	1.35
Detroit Pistons	DET	1.07	0.98	0.76	1.11
Golden State Warriors	GSW	0.98	1.09	1.81	1.00
Houston Rockets	HOU	0.76	1.19	1.17	1.01
Indiana Pacers	IND	1.30	0.99	1.37	1.20
Los Angeles Clippers	LAC	1.36	1.34	1.51	1.01
Los Angeles Lakers	LAL	0.69	1.08	0.32	0.34
Memphis Grizzlies	MEM	1.07	1.47	1.54	1.06
Miami Heat	MIA	1.08	1.17	1.37	1.17
Milwaukee Bucks	MIL	1.06	0.88	1.17	1.04
Minnesota Timberwol	MIN	0.93	1.11	0.70	1.05
New Orleans Pelicans	NOP	0.97	1.00	1.31	1.30
New York Knicks	NYK	1.72	1.48	1.30	1.02
Oklahoma City Thunde	OKC	0.46	1.02	1.59	1.26
Orlando Magic	ORL	0.93	1.32	0.75	0.82
Philadelphia 76ers	PHI	0.32	0.41	1.04	1.16
Phoenix Suns	PHX	0.73	0.90	0.94	1.35
Portland Trail Blazers	POR	1.36	0.90	0.76	1.16
Sacramento Kings	SAC	1.04	0.67	1.07	1.27
San Antonio Spurs	SAS	1.50	0.62	0.87	1.14
Toronto Raptors	TOR	1.30	1.05	1.84	1.33
Utah Jazz	UTA	1.00	1.18	0.86	1.24
Washington Wizards	WAS	1.52	1.26	0.37	0.64

CRRA Results: Regular season results Computed from MV statistics for every pair of teams => Heterogeneous CRRA coefficients

If fitness results from strategic interactions then heterogeneous coefficients are predicted => Prediction applicable to winning in NBA basketball.

Correlations with Performance Measures

	Wins	Losses	W/L%	MOV	ORtg	DRtg	NRtg	MOV/A	ORtg/A	DRtg/A	NRtg/A	Offense	Defense
Off CRRA	0.32	-0.32	0.32	0.31	0.38	-0.05	0.32	0.30	0.37	-0.01	0.30	1.00	
Def CRRA	0.48	-0.48	0.48	0.51	0.35	-0.43	0.50	0.50	0.34	-0.42	0.50	0.07	1.00

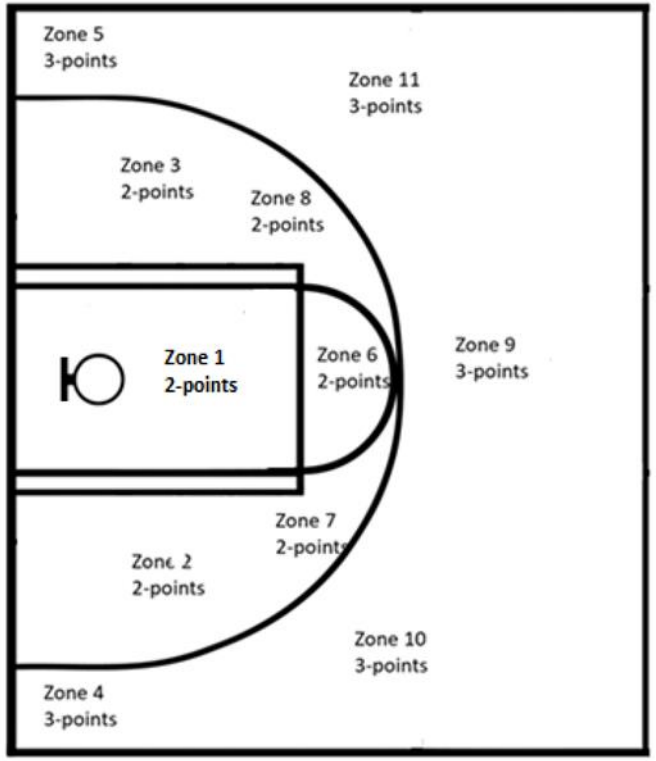
- Critical values: 5% level is 0.349 and 1% is .449 two-tailed test

Table 7: Traditional Performance Analysis and CRRA Coefficients

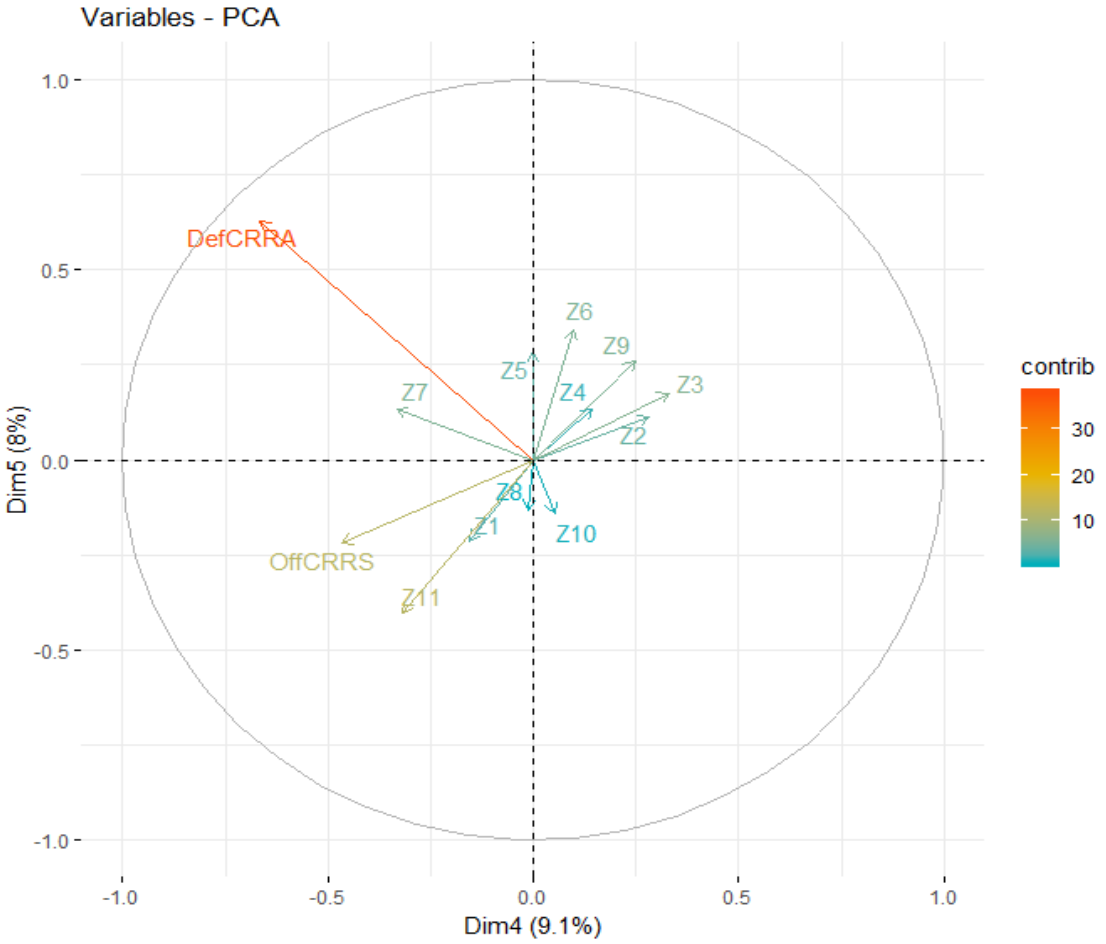
Wins	Total Wins from 82 regular season games
Losses	Total Losses from 82 regular season games
Win/Loss %	Wins/Losses as a percentage
MOV	Margin of Victory; the formula is PTS - Opp PTS.
Ortg	Offensive rating = Estimated points produced (players) or scored (teams) per 100 possessions
DRtg	Defensive rating = Estimate of points allowed per 100 possessions
NRtg	Net Rating (difference)
MOV/A	Margin of Victory adjusted for opponent's strength
Ortg/A	Offensive rating adjusted for opponents strength
DRtg/A	Defensive rating adjusted for opponents strength
NRtg/A	Net Rating adjusted for opponent's strength

What are the Strategic Implications from a high CRRA?

- Conduct a principal component analysis of shots taken from the 11 court zones for the 30 NBA teams plus offensive and defensive team CRRA's
- Both Defensive (shots against) and Offensive Factors (shots taken)
- For Defense the first 6 factors (for factors explaining > 5% of variance) and only 2 of these 6 factors were significantly related to the Win/Loss Ratio and Top Team % performance measures
  - Top factors were not correlated with winning
  - Factors 4 and 5 factor scores by team were both significantly related to winning (Win/Loss Ratio & Top 3 teams)
  - Factors 4 (9% variance) and factor 5 (7% of variance)
- No offensive factors were positively correlated to winning but two were significantly negatively correlated to winning (i.e., bad strategy)
  - Reinforced the defense factor results discussed next

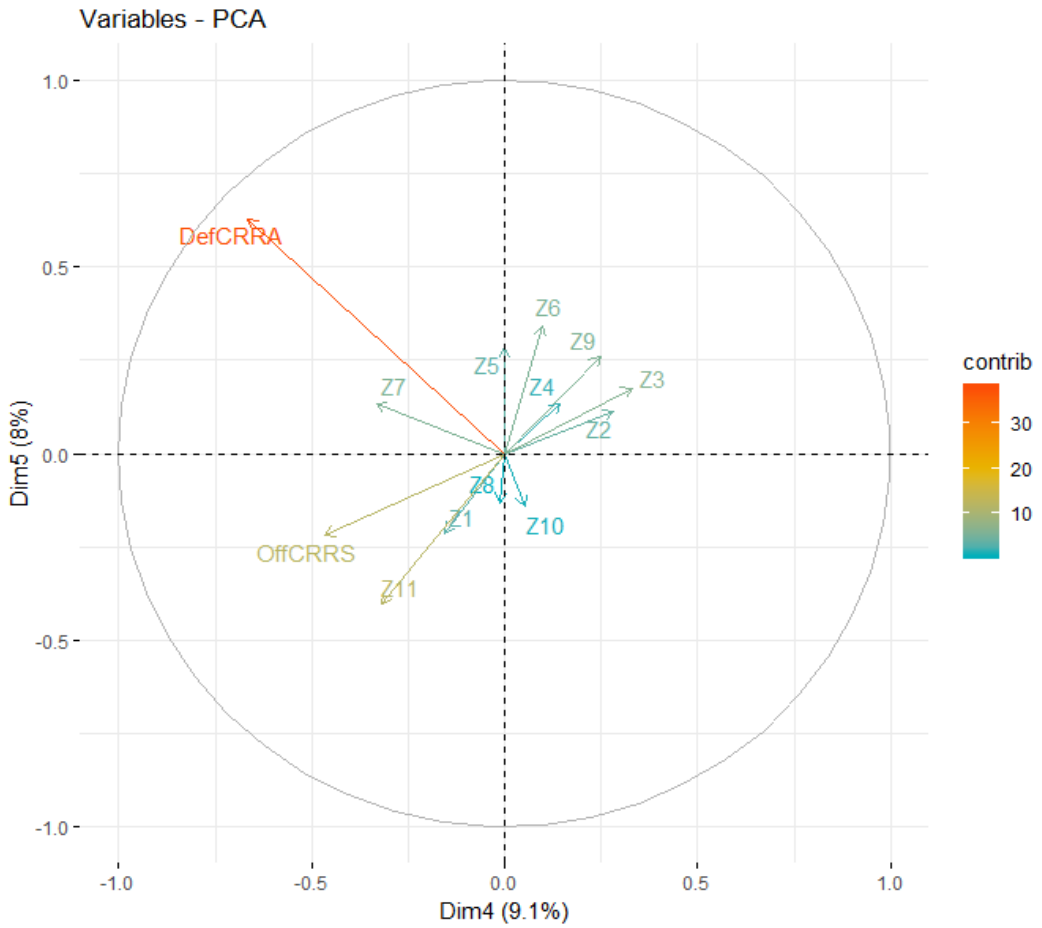


Results: Team's Defensive and Offensive CRRA (i.e., opponent's offensive and defensive CRRA respectively)



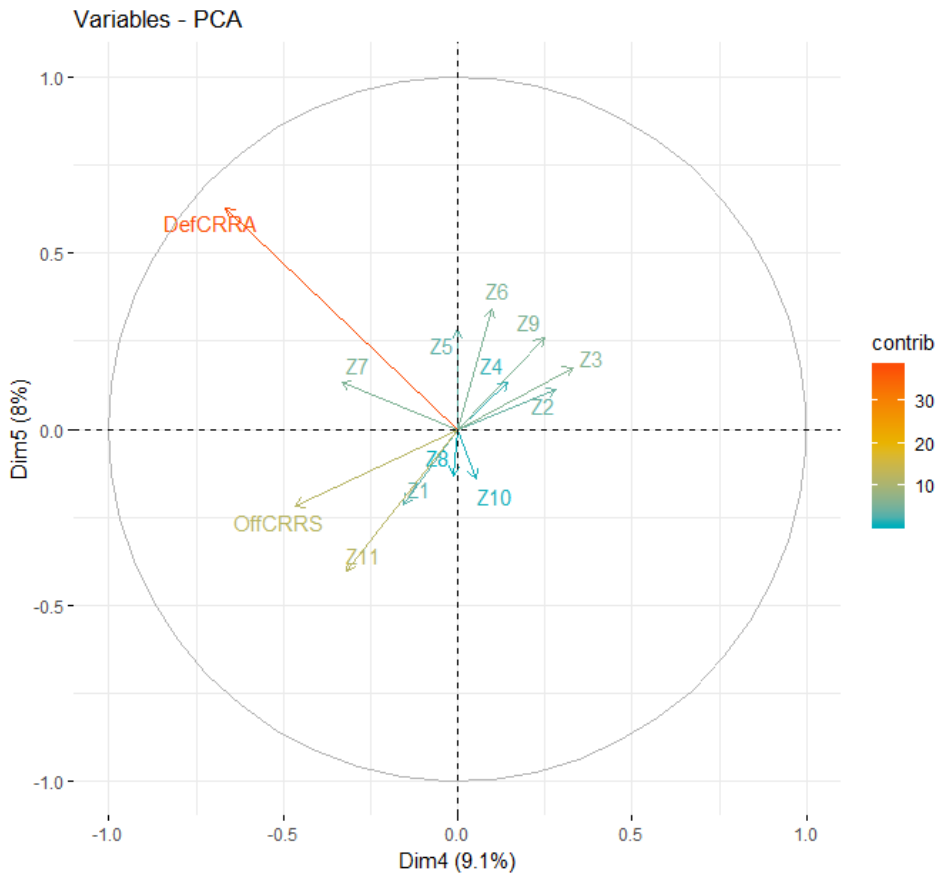
	PC4	PC5
DefCRRA	-0.6129	0.618495
OffCRRS	-0.42928	-0.21449
Z1	-0.14268	-0.20891
Z2	0.259841	0.109715
Z3	0.304059	0.172354
Z4	0.131491	0.132533
Z5	-0.00211	0.279454
Z6	0.089973	0.33685
Z7	-0.3063	0.132499
Z8	-0.01298	-0.12946
Z9	0.227832	0.256766
Z10	0.049475	-0.13859
Z11	-0.29328	-0.39685

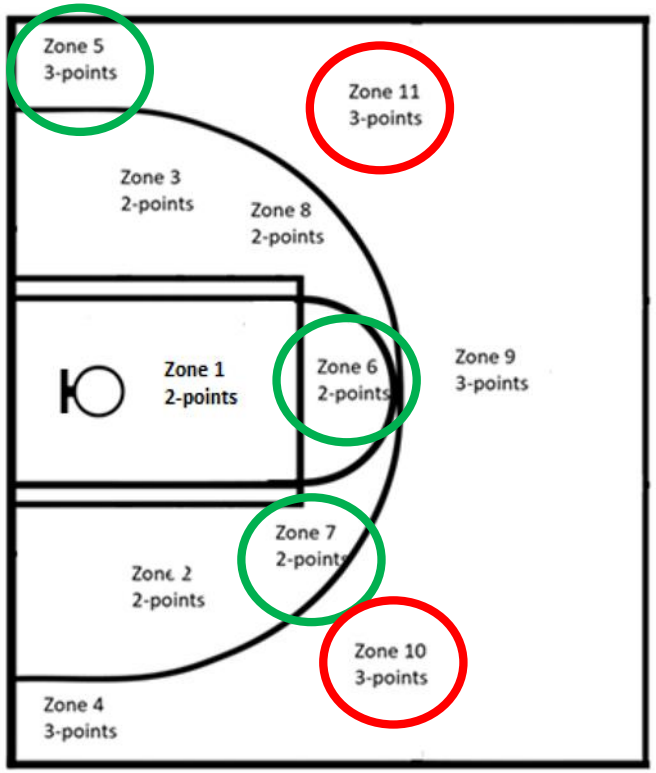
Factors 4 and 5: Offsetting Defensive CRRA



- Correlation
  - Factor 4 negative correlation (r=-.36, -.33 with W/L, Top 3 respectively)
  - Factor 5 positive correlation (r=+.29, +.31) with W/L, Top 3 respectively)
- DefCRRA Factor 4 = -0.61 (weak defense);
- DefCRRA Factor 5 (strong defense) = +0.61

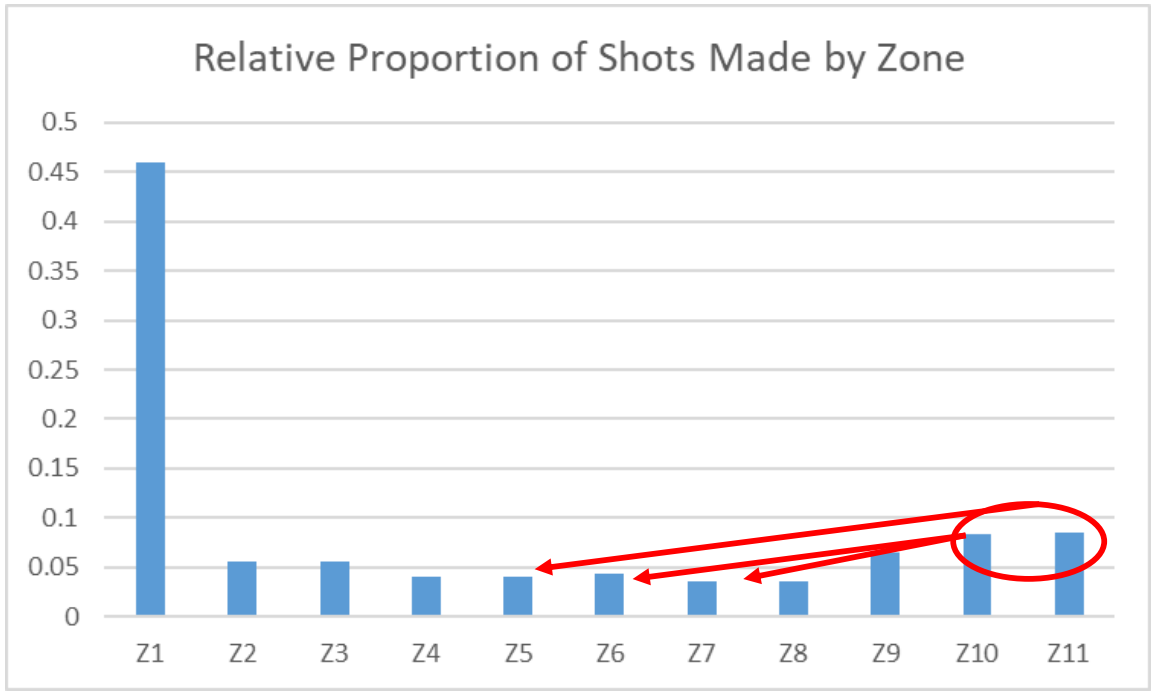
Factors 4 and 5: Offsetting Defensive CRRA

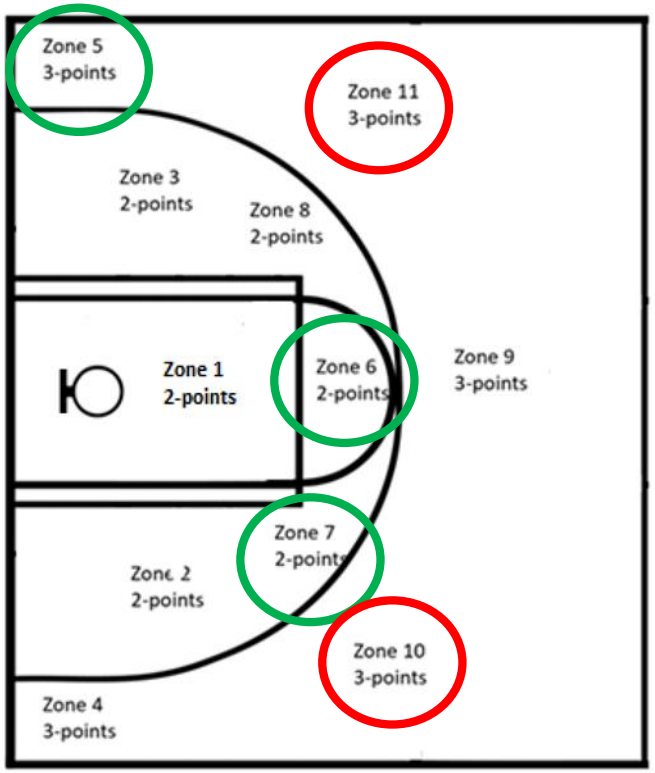




Factors 4 and 5: Offsetting Defensive CRRA

Strong risk averse defense shifts weight away from Z10 & Z11
To Z5, Z6 and Z7





Out of Sample Post Season Equilibrium Predictions

Model: Win/Loss ~ Defensive E(Points Given Up) + Offensive E(Points Made)

	Min	1Q	Median	3Q	Max
Deviance Residuals	-1.86559	-1.04924	-0.02999	1.03794	1.77467

Coefficients	Estimate	Std. Error	z value	Pr(>|z|)
Intercept	-9.492	3.7	-2.565	0.0103*
Defense E(Points Allowed)	-12.196	5.821	-2.095	0.0361*
Offense E(Points Made)	20.182	5.278	3.824	0.0001***

Significance codes: 0 '***' 0.001 '**' 0.01 '*' 0.05

Table 4: Analysis of Equilibrium Strategy Fitness

Preference Analysis of Final Series (GSW versus CLE) --- Simple Dynamics --- Defense induces risk aversion which in turn permits offense to be less risk averse

Final Playoffs *2016/17)	GSW Offensive CRRA/CLE Defensive CRRA	CLE Offensive CRRA/GSW Defensive CRRA
Game 1 (GSW wins)	1.2	3.4
Game 2 (GSW wins)	0.5	3.3
Game 3 (GSW wins)	0.4	3.4
Game 4 (CLE wins)	0.4	3.4
Game 5 (GSW wins)	0.3	3.4

Table 6: CRRA Coefficients from the Final Playoff Series for 2016/17

Conclusions: Preference Dynamics

- Strong defensive teams appear to be able induce higher levels of risk aversion from the opposing offense
  - => greater tradeoff between risk and expected payoffs
  - Evaluating risk coefficients requires both offense and defensive CRRA's e.g., a bad combination is both low!
  - Overall teams are risk averse
- Factor 5 and reinforced in Final Series: If the defense is successful then the offensive can become less risk averse
  - Could be a generic result --- e.g., earlier work with NFL Pittsburgh Steelers (impact of Troy Palamalu)

Thank You!