

Overcoming the incentive incompatibility of tournaments with multiple group stages

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What does incentive compatibility mean in this talk?

Some problems that can undermine the integrity of sport

- ▶ Match fixing
- ▶ Illegal gambling
- ▶ Doping
- ▶ **Tanking: deliberately dropping points or losing a game in order to gain some other advantage**

The issue to be addressed here

- ▶ Ill-constructed rules \Rightarrow a contestant may be *strictly better off* by exerting a lower effort (not only in expected terms)
- ▶ This issue is well discussed in social choice theory since the famous Gibbard-Satterthwaite theorem, but it remains (largely) unexplored on the field of sports
- ▶ The discussion is NOT about strategic manipulation in a probabilistic environment

Outline

- 1 Motivation: a real-world example
- 2 The model
- 3 How can we avoid incentive incompatibility?
- 4 Summary

11th Men's European Handball Championship

What is the EHF Euro?

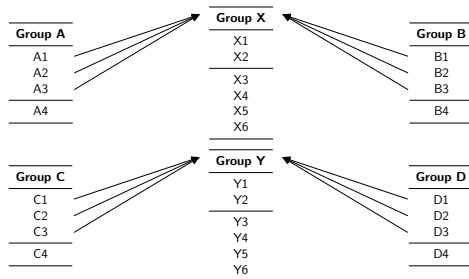
- ▶ The official competition for senior men's national handball teams of Europe
- ▶ It takes place in every second year since 1994
- ▶ The tournament also serves as a qualifier for the Olympic Games and the World Championship

EHF Euro 2014

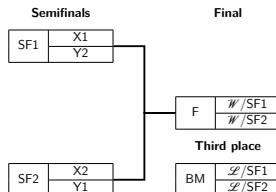
- ▶ It was held in Denmark between 12 and 26 January
- ▶ Preliminary round: 16 national teams are divided into four groups (A-D) to play in a round-robin format
- ▶ The top three teams in each group qualify to the main round
- ▶ Main round: Groups A and B form the first group, Groups C and D form the second group
- ▶ Matches played before are carried over to the main round

The tournament design of EHF Euro

(a) Group stages: preliminary and main rounds



(b) Knockout stage



Ranking rules

Tie-breaking in the group stage

The ranking in each group is determined as follows:

- 1 Higher number of points obtained in all group matches (two points for a win, one point for a draw and no points for a defeat)
- 2 Higher number of points obtained in the group matches played amongst the teams in question
- 3 Superior goal difference from the group matches played amongst the teams in question

In short: head-to-head results (it can be called “UEFA” rule)

Why can a team manipulate?

Because it is interested in qualifying with the teams against which it has obtained the most points.

EHF Euro 2014 – Group C

Match results

Date	First team	Second team	Result
13 January 2014, 18:00	Serbia	Poland	20-19
13 January 2014, 20:15	France	Russia	35-28
15 January 2014, 18:00	Russia	Serbia	27-25
15 January 2014, 20:15	Poland	France	27-28
17 January 2014, 18:00	Poland	Russia	to be played
17 January 2014, 20:15	Serbia	France	to be played

Standing after two matchdays

Pos = Position; W = Won; D = Drawn; L = Lost; GF = Goals for; GA = Goals against; GD = Goal difference; Pts = Points. All teams have played 2 matches.

Pos	Team	W	D	L	GF	GA	GD	Pts
1	France	2	0	0	63	55	8	4
2	Serbia	1	0	1	45	46	-1	2
3	Russia	1	0	1	55	60	-5	2
4	Poland	0	0	2	46	48	-2	0

Analysis of the situation before the last matchday

What should Poland do?

- ▶ It is certainly eliminated if it does not win against Russia
- ▶ It carries over 0 points, 46 goals for and 48 goals against to the main round if it wins against Russia and Serbia plays at least a draw against France
- ▶ If Poland wins by x goals against Russia and Serbia loses, there will be three teams with 2 points, which obtained 2 points in the group matches played among them:
 - ▶ Poland will have a head-to-head goal difference of $x - 1$
 - ▶ Russia will have a head-to-head goal difference of $2 - x$
 - ▶ Serbia will have a head-to-head goal difference of -1
- ▶ Poland will qualify and carry over 2 points to the main round if $x \leq 2$
- ▶ It is unfavourable for Poland to win by more than three goals against Russia

What happened in the group?

- ▶ Poland vs Russia was 24-22 and Serbia vs France was 28-31
- ▶ Poland carried over 2 points to the main round
- ▶ Poland vs Russia was 10-14 after 30 minutes (half-time), while the match stood at 21-16 in the 48th, 22-17 in the 50th, and 23-18 in the 52nd minute

2nd Half

CHRAPKOWSKI P. 11 (32') 14
 LUCAK J. 12 (32') 14
 SYPRZAK K. 13 (34') 14
13 (34') 15 ATMAN P.
 LUCAK J. 14 (35') 15
 CHRAPKOWSKI P. 15 (37') 15
 CHRAPKOWSKI P. 16 (39') 15
 WISNIEWSKI A. 17 (43') 15
17 (44') 16 SHISHKAREV D.
 JURECKI M. 18 (45') 16
 JURECKI M. 19 (46') 16
 LUCAK J. 20 (48') 16
 JASZKA B. 21 (49') 16
21 (49') 17 SHISHKAREV D.
 JASZKA B. 22 (50') 17
22 (51') 18 ZHITNIKOV D.
 JURECKI M. 23 (52') 18
23 (53') 19 IGROPULO K.
23 (53') 20 SKOPINTSEV O.
23 (58') 21 ATMAN P.
 JURECKI B. 24 (60') 21
24 (60') 22 ZHITNIKOV D.

The mathematical framework

Manipulation

A team can *manipulate* in a tournament with multiple group stages if by changing the outcome of some of its matches to a less favourable one, it is guaranteed to gain from this change in the main round, provided that all other match results are fixed.

Strategy-proofness (SP)

A tournament with multiple group stages is called *strategy-proof* or *incentive compatible* if there exists no set of results under which a team can manipulate.

Natural assumptions

- ▶ Groups are standard single or double round-robin tournaments
- ▶ All rankings are monotonic: ranking rules in the preliminary and main round groups, as well as the qualification rule
- ▶ Matches in the main round are not replayed if the two teams have already played against each other

The main result

Theorem

A tournament with multiple group stages does not satisfy strategy-proofness if the following conditions hold:

- ▶ There exist two teams in the same preliminary and main round groups
- ▶ At least one team is eliminated from this preliminary round group

How can incentive compatibility be guaranteed?

- 1 Teams qualifying from the same preliminary round group should be drawn into different main round groups (UEFA Champions League 1999-2003): no results are carried over
- 2 All results (or a monotonic transformation of them) are carried over to the main round group, independently of the set of teams advanced

Handball tournaments with multiple group stages

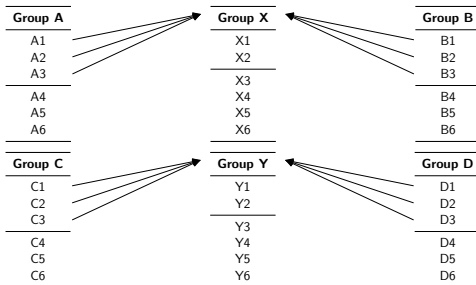
Tournament	Year(s)	Type	Preliminary round			Main round		
			Gr.	Teams	Q	Gr.	Teams	Q
EHF Euro Men	2002–	S	4	4	3	2	6	2
EHF Euro Women	2002–	S	4	4	3	2	6	2
EHF Women's CL	2013/14–	D	4	4	3	2	6	4
IHF World Men	2003	S	4	6	4	4	4	1
IHF World Men (G66)	2005, 2009-2011, 2019–	S	4	6	3	2	6	2
IHF World Men (G46)	2007	S	6	4	2	2	6	4
IHF World Women (G66)	2003-2005, 2009	S	4	6	3	2	6	2
IHF World Women (G46)	2007	S	6	4	2	2	6	4

Notes: S = single round-robin (in groups); D = double round-robin (in groups); Gr. = Number of groups in the preliminary and main round, respectively; Teams = Number of teams in each group of the preliminary and main round, respectively; Q = Number of teams qualified from each group of the preliminary and main round, respectively

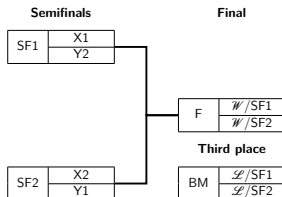
Abbreviations: EHF Euro Men (Women) = European Men's (Women's) Handball Championship; EHF Women's CL = Women's EHF Champions League; IHF World Men (Women) = IHF World Men's (Women's) Handball Championship

Tournament design G66

(a) Group stages: preliminary and main rounds

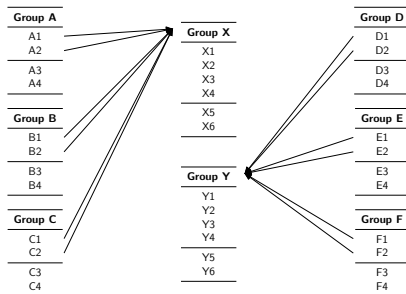


(b) Knockout stage

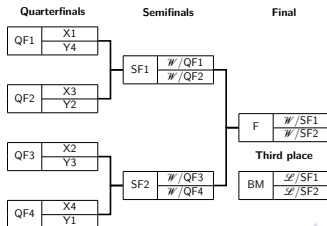


Tournament design G46

(a) Group stages: preliminary and main rounds



(b) Knockout stage



The effects of incentive compatible rules

Two strategy-proof versions of this tournament design

- ▶ All preliminary round results are carried over
- ▶ Only half of the points from the preliminary round results are carried over (this idea comes from the Belgian First Division A)

Implementation (based on Csató [2019], forthcoming in ITO)

- ▶ They are compared through simulations with the traditional (incentive incompatible) version of designs G66 and G46

Tournament metrics

- ▶ The average pre-tournament ranks of the clubs in the Final Four, that is, the winner, the second-, third-, and fourth-placed teams
- ▶ The expected quality of all matches, measured by the sum of the playing teams' pre-tournament ranks
- ▶ The expected competitive balance of all matches, measured by the difference between the playing teams' pre-tournament ranks

The simulation

Playing abilities

- ▶ The probability with which team i defeats team j is fixed *a priori*:

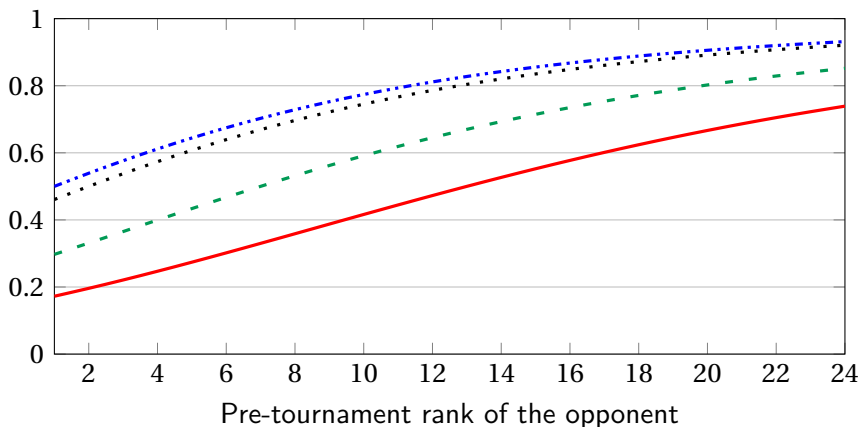
$$p_{ij} = \frac{1}{1 + [(i + \beta)/(j + \beta)]^\alpha}$$

- ▶ $1 \leq i, j \leq 24$ is the identifier of the teams
- ▶ $\alpha, \beta \geq 0$ are parameters (baseline: $\alpha = 4$, $\beta = 24$)

Group formation

- ▶ The teams should be drawn into k groups before the start of the tournament
- ▶ *Seeded* version: the strongest k teams are placed in Pot 1, the next strongest k teams in Pot 2, and so on
- ▶ *Unseeded* version: fully random seeding

The probability that team i beats its opponent (baseline, $\alpha = 4$, $\beta = 24$)

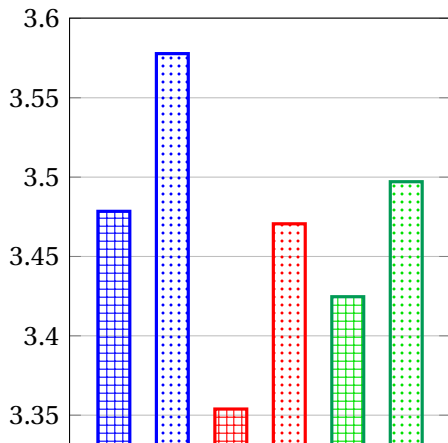


The tournament designs of the simulation

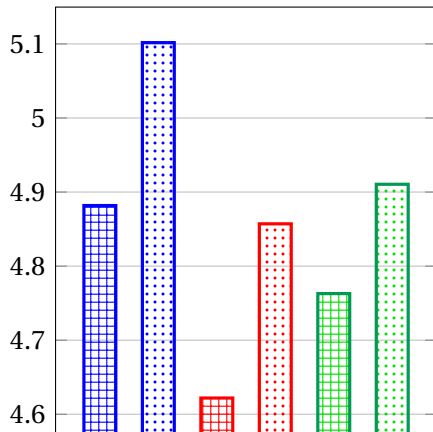
Notation	Format	Seeding policy	Description
G66/S	G66	seeded	original incentive incompatible
G66/R	G66	unseeded	original incentive incompatible
G66◇/S	G66	seeded	all points are carried over
G66◇/R	G66	unseeded	all points are carried over
G66★/S	G66	seeded	half of all points are carried over
G66★/R	G66	unseeded	half of all points are carried over
G46/S	G46	seeded	original incentive incompatible
G46/R	G46	unseeded	original incentive incompatible
G46◇/S	G46	seeded	all points are carried over
G46◇/R	G46	unseeded	all points are carried over
G46★/S	G46	seeded	half of all points are carried over
G46★/R	G46	unseeded	half of all points are carried over

The average pre-tournament rank I. (G66)

Average rank of #1



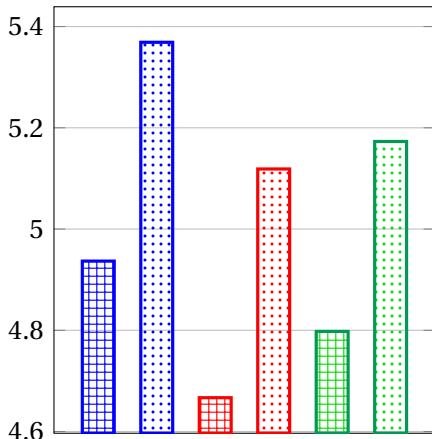
Average rank of #2



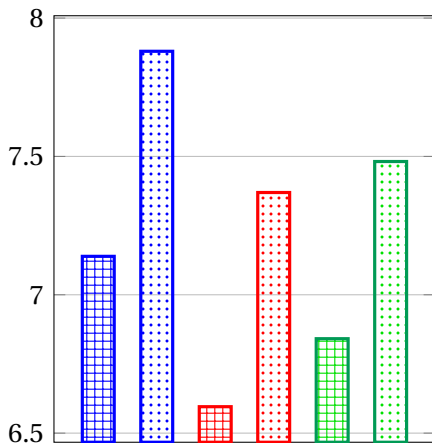
■ G66/S
 ■ G66/R
 ■ G66◇/S
 ■ G66◇/R
 ■ G66★/S
 ■ G66★/R

The average pre-tournament rank II. (G66)

Average rank of #3



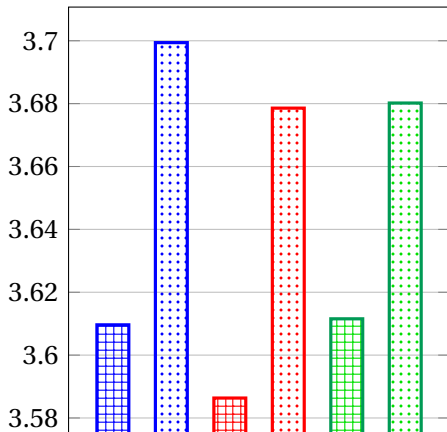
Average rank of #4



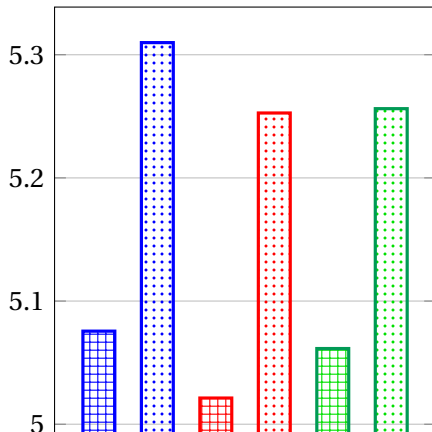
■ G66/S
 ■ G66/R
 ■ G66◇/S
 ■ G66◇/R
 ■ G66★/S
 ■ G66★/R

The average pre-tournament rank I. (G46)

Average rank of #1



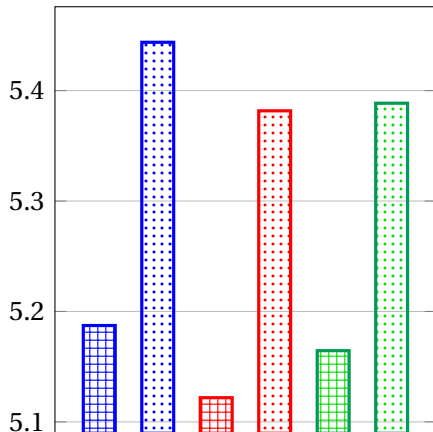
Average rank of #2



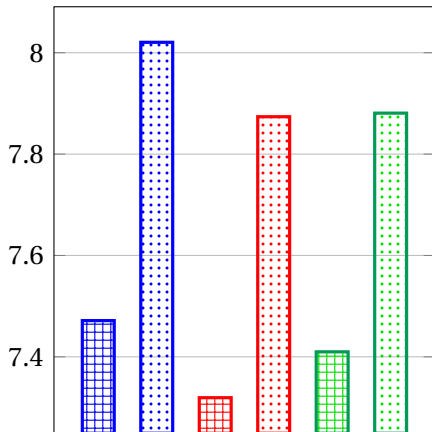
■ G46/S
 ■ G46/R
 ■ G46◇/S
 ■ G46◇/R
 ■ G46★/S
 ■ G46★/R

The average pre-tournament rank II. (G46)

Average rank of #3



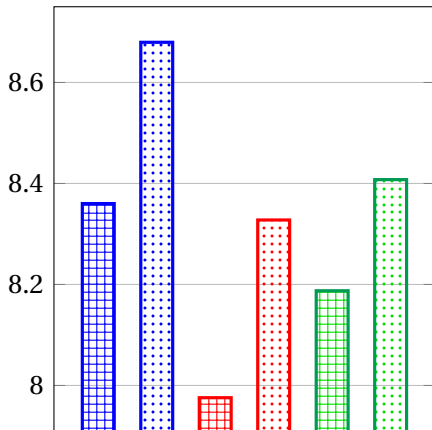
Average rank of #4



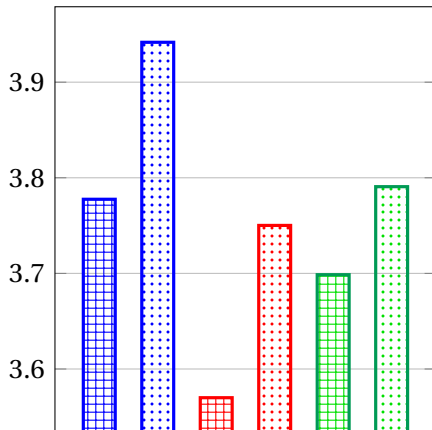
■ G46/S
 ▨ G46/R
 ■ G46◇/S
 ▨ G46◇/R
 ■ G46★/S
 ▨ G46★/R

Characteristics of the tournament final (G66)

Expected quality



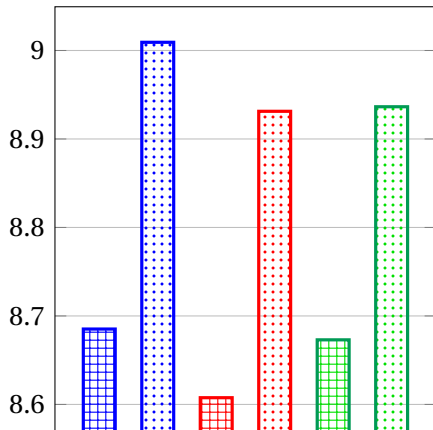
Expected competitive balance



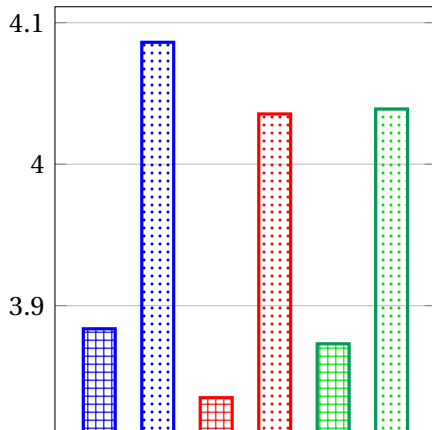
■ G66/S
 ■ G66/R
 ■ G66◇/S
 ■ G66◇/R
 ■ G66★/S
 ■ G66★/R

Characteristics of the tournament final (G46)

Expected quality



Expected competitive balance



■ G46/S
 ■ G46/R
 ■ G46◇/S
 ■ G46◇/R
 ■ G46★/S
 ■ G46★/R

Conclusions

Main findings

- ▶ It is proved that multi-stage tournaments are incentive incompatible when some results are carried over
- ▶ A similar situation occurred during the EHF Euro 2012: Slovenia should have won against Iceland for qualification to the main round, but it would be better not to win by more than 3 goals in order to carry over its result against Iceland
- ▶ Men's and Women's European Handball Championships are organized according to this design since 1994
- ▶ Tournament with multiple group stages are widely used in basketball, cricket, handball, or volleyball

Our suggestion: carry over the half of all points from the groups of the preliminary round (regardless that some matches were played against teams already eliminated from the tournament)

“Historical correctness is a secondary consideration; a case invented might also serve the purpose as well, only historical ones are always to be preferred, because they bring the idea which they illustrate nearer to practical life.”

(Carl von Clausewitz: *Vom Kriege*)

Thank you for your attention!