Continuous-time corporate finance and expected equity returns

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Session overview

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Session objective

- The session aims to provide an introduction to modeling techniques in continuous-time corporate finance as well as explore the link between corporate finance and expected equity returns (beta)
- Applications include
  - Irreversible investment
  - Capital structure and credit risk
  - The relationship between corporate finance and equity returns
  - Game-theoretical aspects of real options exercise (time permitting)

Part I

Introduction to continuous-time corporate finance: Capital investment under uncertainty

General framework

- In continuous-time (dynamic) corporate finance, the value of a claim F is typically represented as a function of the value of some underlying economic variable
- Examples of pairs of claims and underlying variables include
  - Equity and the value of an unlevered firm
  - A liquidation option and the operating cash flow
  - An option to drill and the price of crude oil
- We will generally denote an underlying stock variable (e.g., firm value) by V and a flow variable (e.g., project cash flow) by x

General framework

- The underlying variable (let's use V here, the logic for x is analogous) is usually assumed to follow a geometric Brownian motion (GBM)
$$dV_t = \mu V_t dt + \sigma V_t dz_t, \tag{1}$$
where parameter μ denotes the deterministic drift rate, σ is the instantaneous standard deviation of returns on V, and dz is an increment of a Wiener process
- In most problems, the (deterministic) riskless interest rate r and the drift rate μ have to satisfy $\mu < r$ so that finite valuations and finite exercise times can be obtained

General framework

- The instantaneous payoff to the holder of the claim is π
- In a typical application that we will focus on, one of the following assumptions is made
  - The value of the claim can be spanned by a portfolio of traded assets
  - All involved parties are risk neutral
- Consequently, riskless interest rate can be conveniently used to discount all cash flows
- Given the above assumptions, the value of claim F has to satisfy the following relationship

$$rFdt = \pi dt + E\left[dF\right] \tag{2}$$

General framework

- Equation (2) means that the expected return on claim F over the time interval dt equals the riskless rate r times dt
- The expected return consists of the deterministic payout πdt and the expected capital gain $E[dF]$
- Expanding the RHS of (2), applying Itô's lemma, and dividing both sides of the equation by dt results in the following differential equation:

$$rF = \underbrace{\pi}_{\text{Payout flow}} + \underbrace{\mu V \frac{\partial F}{\partial V} + \frac{1}{2}\sigma^2 V^2 \frac{\partial^2 F}{\partial V^2} + \frac{\partial F}{\partial t}}_{\text{Expected instantaneous capital gain}} \tag{3}$$

General framework

- The value of claim F is determined by solving equation (3) subject to appropriate boundary conditions
  - In all but one cases, we will consider infinite horizon problems, for which $\frac{\partial F}{\partial t} = 0$
- Analytical solutions exist only to very special cases of (3), such as
  - European call and put options
  - American perpetual call and put options
- In many applications, the claim value F is jointly determined with the optimal timing of an action that the claimholder is entitled to undertake
  - such as default, growth option exercise etc

Example: The standard model

- The general framework discussed can be illustrated with the standard investment model
  - as described by McDonald and Siegel (1986), and extensively analyzed in Dixit and Pindyck (1994)
- The basic problem is to find the value of a growth option and the optimal timing of irreversible investment associated with cost I, given that the value of the investment project follows the familiar geometric Brownian motion (1):

$$dV_t = \mu V_t dt + \sigma V_t dz_t$$

- The firm is risk-neutral and maximizes the value of the investment option, $F(V)$, by choosing the threshold value of V at which the project is undertaken

The standard model

- Since there is no payout to the holder of the option, the option value satisfies

$$rFdt = E\left[dF\right]$$

- As the option does not have a finite maturity, the resulting (ordinary) differential equation is simply:

$$rF = \mu V \frac{\partial F}{\partial V} + \frac{1}{2}\sigma^2 V^2 \frac{\partial^2 F}{\partial V^2} \tag{4}$$

The standard model

- The general solution to equation (4) has the following form:

$$F(V) = A_1 V^{\lambda_1} + A_2 V^{\lambda_2} \tag{5}$$

where A_1 and A_1 are constants, and

$$\lambda_{1,2} = -\frac{\mu}{\sigma^2} + \frac{1}{2} \pm \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}}$$

- The existence (and the relative simple form) of the analytical solution is thanks to the absence of instantaneous payout and to the infinite horizon ($\frac{\partial F}{\partial t} = 0$)
- It can be verified that $\lambda_1 > 1$ and $\lambda_2 < 0$

Digression #1: Value matching and smooth pasting

- For reasons described in Dixit and Pindyck (1994), and more rigorously in Dumas (1991), the optimality of an *irreversible* (or a partially reversible) decision is ensured by the smooth-pasting condition, which is equivalent to the equality of the first-order derivatives of the claim value at the decision threshold
- If a decision affects the value of more than one claim (i.e., equity and debt), the claims values which are not maximized are generally not *differentiable* at a decision threshold
  - For that reason debt value exhibits a kink at a bankruptcy threshold (we will discuss it in the next section)

Value matching and smooth pasting

- In adddition, all claim values must be *continuous* at the decision threshold (value matching)
- We will see later that the analogous rules are different at a fully *reversible* decision threshold

The standard model

- In order to find the value of the investment option, $F(V)$, and the optimal investment threshold, $\overline{V}$, the following boundary conditions are applied to (5):

$$F\left(\overline{V}\right) = \overline{V} - I \tag{6}$$

$$\left.\frac{\partial F(V)}{\partial V}\right|_{V=\overline{V}} = 1 \tag{7}$$

$$F(0) = 0 \tag{8}$$

- Conditions (6) and (7) are the value-matching and smooth-pasting conditions, respectively, and ensure continuity and differentiability of the value function at the investment threshold

- Condition (8) ensures that the investment option is worthless at $V = 0$ and implies that $A_2 = 0$

The standard model

- Substitution of (5) into (6)-(8) and some algebraic manipulation yield the optimal investment threshold:

$$\overline{V} = \frac{\lambda_1}{\lambda_1 - 1} I$$

- Since $\lambda_1 > 1$, the optimal investment threshold is strictly larger than I (cf. the NPV rule, which merely requires that $\overline{V} \geq I$)

The standard model

- The mark-up $\lambda_1/(\lambda_1 - 1) > 1$ reflects the value of waiting associated with the uncertainty of the project's value and the irreversibility of the investment decision
- The value of the option to invest, $F(V)$, is given by

  $$F(V) = \left(\overline{V} - I\right)\left(\frac{V}{\overline{V}}\right)^{\lambda_1}$$

  where $\overline{V} - I$ is the NPV of the project at the moment of undertaking the investment
- The second factor is the probability-weighted discount factor, which reflects the present value of \$1 received when the cash flow process hits the investment threshold $\overline{V}$ when starting from V

The standard model

- The value of the optimal investment threshold is positively related both to the volatility of the project's value as well as to its growth rate
  - I.e., the higher σ and μ are, the higher V must be reached for the project to be undertaken
- $F(V)$ increases with the volatility of the value of the project
  - λ_1 is a decreasing function of σ and F itself is decreasing with λ_1

Summary

- In dynamic corporate finance, the value of a claim is a function of the value of some underlying asset, which typically follows a geometric Brownian motion
- Therefore, we can demonstrate that a claim value is a solution to a differential equation
  - In most applications, we will consider infinite horizon problems, for which the differential equation is ordinary and its analytical solution tractable
- In many situations, the value of a claim is determined jointly with the optimal action of the claimholder(s)
- The optimal timing of a lumpy irreversible investment under uncertainty is a first example of our framework in action

Part II

Capital structure and credit risk

Introduction

Outline

- Leland (1994) as an example of a first-passage time model
  - Building on the early structural model of credit risk (Merton, 1974)
  - Introduction of the infinite horizon leading to greater analytical tractability
- Strategic debt service (Mella-Barral and Perraudin, 1997; Fan and Sundaresan, 2000) and other extensions

Key papers

- Leland, H. E. (1994): "Corporate Debt Value, Bond Covenants, and Optimal Capital Structure," Journal of Finance, 49, 1213–1252
- Davydenko, S. A., and I. A. Strebulaev (2007): "Strategic Actions and Credit Spreads: An Empirical Investigation," Journal of Finance, 62(6), 2633–2671

Structural models of credit risk

- One of the earliest applications of the presented dynamic valuation methodology in corporate finance is the assessment of credit risk
- Structural (or market-based/firm value-based) models of credit risk generally assume that market participants can observe firm dynamics (as given by equation (1))
  - Debt and equity of the firm can be valued as claims contingent of the firm's value
- The structural modeling of credit risk is a modern alternative to reduced-form, intensity-based approach and has a number of advantages over more traditional accounting-based methods:
  - Forward-looking approach and no information lags
  - Reliance on market values (next to accounting numbers)
  - Asset volatility explicitly accounted for

First-passage time models

- In first-passage time models, default can be triggered by the value of assets falling below a certain value at any time before the maturity
  - this value can be a function of time (Black and Cox, 1976)
  - this value can be endogenous, e.g., chosen by equityholders (Leland, 1994)
  - payoff to debt- and equityholders upon default may be an outcome of a bargaining process (Mella-Barral and Perraudin, 1997)
- In general, the basic model structure has a similar characteristics to that of Merton-Black-Scholes model
- The discussion of Leland (1994) will serve as an example of a contemporary first-passage time model

Leland (1994)

Major findings

- Closed-form results for the value of corporate debt and optimal capital structure
- Relationship between credit spread and firm's assets volatility, bankruptcy costs and riskless interest rate
- Differences in behavior of junk vs. investment grade bonds
- Optimal level of leverage

Leland (1994)

Assumptions

- Bankruptcy determined endogenously by the firm's inability to raise equity capital
- Time-independence allowing for closed-form results:
  - approximation of a long maturity debt
  - rolling-over a short term debt at a fixed interest rate
- Activities of the firm unchanged by capital structure
- Static capital structure (once chosen)

Set-up of the model

- Value of the firm's assets (in the risk-neutral world) follows a diffusion process

$$dV_t = rV_t dt + \sigma V_t dz$$

which is the same as in Merton (1974)

- As there is no fixed horizon, the resulting differential equation does not have a time derivative and the value of security F that receives instantaneous payout c equals to

$$F(V) = \frac{c}{r} + A_1 V + A_2 V^{-X} \tag{9}$$

where $X = \frac{2r}{\sigma^2}$

- Note that $\lambda_1^{\mu=r} = 1$ and $\lambda_2^{\mu=r} = -X$

Set-up of the model

- Debt is assumed to pay perpetual coupon b
- Bankruptcy costs equal αV, where $\alpha \in [0, 1]$
- Value of debt, D, is determined by applying boundary conditions to (9) and substituting b for instantaneous payout c

$$
\begin{aligned}
D(V_B) &= (1-\alpha) V_B \\
\lim_{V \to \infty} D(V) &= \frac{b}{r}
\end{aligned}
$$

The value of debt

- The value of debt is therefore equal to

$$D(V) = \frac{b}{r} + \underbrace{\left((1-\alpha)V_B - \frac{b}{r}\right)}_{\text{PV loss}} \underbrace{\left(\frac{V}{V_B}\right)^{-X}}_{\text{Discount factor}}$$

- The discount factor reflects the PV of \$1 received upon hitting default trigger V_B when starting from V
  - A similar discount factor appeared in the standard model of investment
  - Here, it is associated with a *downward* movement of the GBM (so the root λ_2 is relevant); recall that for $\mu = r$ used in Leland (1994), $\lambda_2 = -X$

Bankruptcy costs

- The PV of bankruptcy costs, *BC*, satisfies the following two conditions

$$\begin{aligned} BC(V_B) &= \alpha V_B \\ \lim_{V\to\infty} BC(V) &= 0 \end{aligned}$$

- Therefore

$$BC(V) = \alpha V_B \left[V/V_B\right]^{-X}$$

Tax shield

- Denote tax rate by τ
- Then, instantaneous tax benefits equal τb
- Denote the PV of tax benefits by TB and observe that

$$\begin{aligned} TB(V_B) &= 0 \\ \lim_{V\to\infty} TB(V) &= \frac{\tau b}{r} \end{aligned}$$

- Then

$$TB(V) = \frac{\tau b}{r}\left(1 - [V/V_B]^{-X}\right)$$

The value of equity

- Since the value of the firm equals

$$v(V) = V - BC(V) + TB(V)$$

the value of equity is

$$\begin{aligned} E(V) &= v(V) - D(V) \\ &= V - (1-\tau)\frac{b}{r} + \left[(1-\tau)\frac{b}{r} - V_B\right][V/V_B]^{-X} \end{aligned}$$

Default trigger

- Default is declared at the point optimal for equityholders, i.e., by setting

$$\left.\frac{\partial E}{\partial V}\right|_{V=V_B} = 0$$

This yields

$$V_B = (1-\tau)\,\frac{b}{r}\,\frac{X}{X+1} = (1-\tau)\,\frac{b}{r+0.5\sigma^2} < (1-\tau)\,\frac{b}{r}$$

Credit spread

- The calculation of the spread for Leland (1994) is performed by observing the the yield y on the bond is defined as

$$D(V) = \frac{b}{y} = \frac{b}{r+s}$$

- The yield spread s therefore equals

$$s = \frac{b}{D(V)} - r = r\frac{k\left(\frac{b}{V}\right)^X}{1-k\left(\frac{b}{V}\right)^X}$$

where

$$k = \frac{1+X-(1-\alpha)(1-\tau)X}{1+X}\left(\frac{(1-\tau)X}{r(1+X)}\right)^X$$

Credit spread

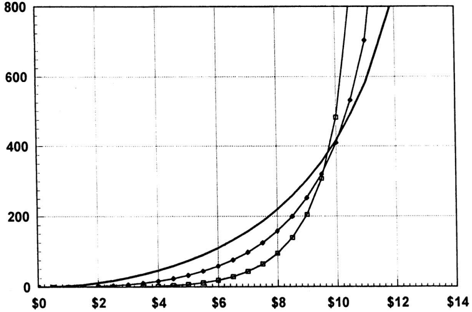


Figure 4. Yield spreads on unprotected debt as a function of the coupon. The lines plot the yield spread, *YS* (in basis points/year), the amount the firm's debt yield exceeds the risk-free rate, as a function of the coupon, C, for varying levels of asset volatility, σ: 15 percent (*open square*), 20 percent (*filled diamond*), and 25 percent (*solid line*). It is assumed that the risk-free interest rate $r = 6.0$ percent, bankruptcy costs are 50 percent ($\alpha = 0.5$), and the corporate tax rate is 35 percent ($\tau = 0.35$).

Credit spread

- It is shown that the yield spread:
  - increases in coupon, b
  - increases in asset volatility, σ, for high V and decreases for $V \to V_B$ ("junk bonds")
  - decreases in risk-free rate, r
  - increases in bankruptcy costs, α
  - decreases in tax rate, τ
  - decreases in asset value, V

Optimal leverage

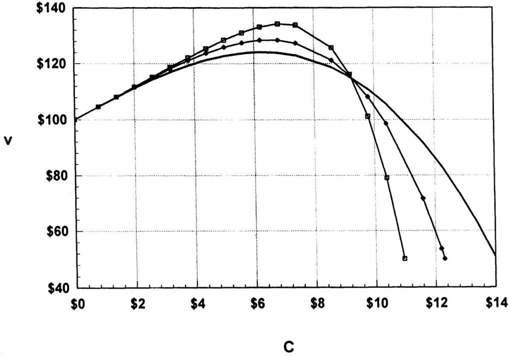


Figure 6. Total firm value as function of the coupon, when debt is unprotected. The lines plot total firm value, v, at varying coupon levels, C, for three levels of asset volatility, σ: 15 percent (*open square*), 20 percent (*filled diamond*), and 25 percent (*solid line*). It is assumed that the risk-free interest rate $r = 6.0$ percent, bankruptcy costs are 50 percent ($\alpha = 0.5$), and the corporate tax rate is 35 percent ($\tau = 0.35$).

Optimal leverage

- Optimal leverage, corresponding to coupon $b^* (V)$, is determined by maximizing $v(V)$ with respect to b
- Optimal leverage:
  - decreases with asset volatility, σ
  - increases with risk-free rate, r
  - decreases with bankruptcy costs, α
  - increases with tax rate, τ

Calibration of the model

- Similar as in Merton (1974), it is the value of equity E, equity volatility σ_E, interest rate r, and coupon rate that is observable
- Again, asset volatility σ, and the value of the firm V, are assumed to be unknown
- The two latter are determined by solving the system of equations linking the values and volatilities of the firm and its equity

Extensions

- To conclude the discussion of the key models of credit risk, the following two extensions will be introduced:
  - Payout from the asset
  - Strategic debt service (Mella-Barral and Perraudin, 1997), also extended for
    - Continuous distribution of bargaining power (Fan and Sundaresan, 2000)
    - Positive likelihood of renegotiation failure (Favara, Schroth, and Valta, 2012)

Payout from the asset

Leland (1994)

- In the original Leland (1994) paper, the elasticity of the probability weighted discount factor to the changes in V is given by $-2r/\sigma^2$
- One can allow for a positive net cash flow out of the firm (at rate $\delta \equiv r - \mu$), so that λ_2 is used instead of $-X$

Payout from the asset

Leland (1994)

- The value of debt (with perpetual coupon b) equals

$$D(V) = \frac{b}{r}\left[1 - \left(\frac{V}{V_B}\right)^{\lambda_2}\right] + (1-\alpha)\, V_B \left(\frac{V}{V_B}\right)^{\lambda_2}$$

where

$$V_B = \frac{-\lambda_2}{1-\lambda_2}\,\frac{b\,(1-\tau)}{r}$$

is equityholders' optimal default trigger

Strategic debt service

Continuous distribution of bargaining power

- Assume that there are no taxes so strategic debt service does not influence the value of the firm
- Let φ^* be the outcome of the Nash bargaining process, being equal to the fraction of the firm received by the shareholders
- Consequently, equityholders receive $\varphi^* V$ and debtholders get $(1 - \varphi^*) V$
- The outside options of equityholders and debtholders are zero and $(1 - \alpha) V$, respectively

Strategic debt service

Continuous distribution of bargaining power

- For $\eta \in [0,1]$ being the bargaining power coefficient of shareholders, the solution of the bargaining game can be written as follows

$$\begin{aligned}\varphi^* &= \arg\max_{\varphi} \left[(\varphi V)^{\eta} \left((1-\varphi) V - (1-\alpha) V \right)^{1-\eta} \right] \\ &= \eta \frac{V - (1-\alpha) V}{V} = \alpha\eta\end{aligned}$$

Strategic debt service

- Assuming that upon strategic default creditors receive fraction $1-\varphi$ of the firm, the value of debt equals

$$D_S(V) = \frac{b}{r}\left[1-\left(\frac{V}{V_S}\right)^{\lambda_2}\right]+(1-\varphi)V_S\left(\frac{V}{V_S}\right)^{\lambda_2}$$

- Recall that there are no taxes (i.e., $\tau = 0$)
  - A strictly positive tax rate requires that we make additional assumptions about the value of tax shield in the renegotiation region
  - In general, the value of the firm becomes endogenous in the choice of the strategic default trigger when $\tau > 0$
- For a detailed exposition of the strategic debt service model with taxes, see Fan and Sundaresan (2000)

Strategic debt service

- The strategic default trigger V_S, the fraction of the firm that accrues to equityholders φ, and coupon payment in the renegotiation region $b_S(V)$ are

$$\begin{aligned} V_S &= \frac{-\lambda_2}{1-\lambda_2}\frac{b}{r(1-\alpha\eta)} \geq V_B \\ \varphi &= \alpha\eta \\ b_S(V) &= (1-\alpha\eta)\delta V \end{aligned}$$

Strategic debt service

Positive likelihood of renegotiation failure

- For their empirical study, Favara, Schroth, and Valta (2012) extend the strategic debt service model to allow for a positive likelihood of renegotiation failure
  - In an otherwise standard set-up, they assume that renegotiation, if attempted, will only succeed with a certain probability, $(1 - q)$
  - Otherwise (with probability q), the renegotiation attempt is unsuccessful, standard default occurs and APR is upheld
- Allowing for a positive likelihood of renegotiation failure results in the modified expression for the optimal strategic default threshold

$$V_S^q = \frac{-\lambda_2}{1-\lambda_2} \frac{b}{r(1-(1-q)\alpha\eta)} \in (V_B, V_S)$$

Empirical proxies in the strategic debt service model

- The distribution of bargaining power and the probability of strategic debt service occurring upon default is generally not observable
- In empirical applications, it can be captured by the following proxies (cf. Davydenko and Strebulaev, 2007):
  - Relative bargaining power:
    - managerial shareholdings
    - size of the firm
    - age of the firm
  - Renegotiation frictions:
    - number of creditors (number/Herfindahl index of bond issues)
    - fraction of public (vs. private) debt
    - fraction of short-term (vs. long-term) debt

Further extensions

- Features that have been incorporated as extensions of the discussed structural models include
  - Stochastic interest rate (Longstaff and Schwartz, 1995)
  - Asymmetric information (Duffie and Lando, 2001)
  - Jumps in asset prices (Zhou, 2000)
  - Separation of ownership and control (Morellec, 2004)
  - Correlated defaults (Zhou, 2001)
  - Cash holdings (Acharya, Davydenko and Strebulaev, 2012)

Summary

- The early structural model of credit risk (Merton, 1974) is based on the well-known European call option formula
- First-passage time models are more realistic as they allow for default before debt maturity
  - Introduction of the infinite horizon leading to greater analytical tractability and the possibility of endogenizing default (Leland, 1994)
- Further extensions of standard first-passage model include strategic debt service and payout from assets
- Any structural model can be used to price debt, estimate default probabilities and calculate credit spreads based on the (observed) firm dynamics

Part III

Corporate finance and equity returns

Introduction

Key papers

- Carlson, M., A. Fisher, and R. Giammarino (2004): “Corporate Investment and Asset Price Dynamics: Implications for the Cross-Section of Returns,” Journal of Finance, 59(6), 2577–2603
- Novy-Marx, R. (2011): “Operating Leverage,” Review of Finance, 15(1), 103–134
- Favara, G., E. Schroth, and P. Valta (2012): “Strategic Default and Equity Risk Across Countries,” Journal of Finance, 67(6), 2051–2095

Overview

- The topic of this session is how corporate finance can affect asset prices
- Many asset pricing models assume that asset return distributions are somewhat given
- Assuming that it is the firm's cash flows (revenues) that are subject to a certain source of risk, can we say anything about the random behavior of claims on the firm's cash flows (such as equity)?
  - The fact that the systematic risk of the firm's cash flow is constant (and known) does not necessarily imply that straightforward conclusions can be drawn about the systematic risk of the firm's equity

Overview

- To address this question, we will focus on the effect of
  - operating leverage (fixed operating costs)
  - growth options
  - financial leverage (debt)
  - an option to default

  on systematic risk of the firm's equity
- Some related literature will be (briefly) mentioned towards the end of the session

Benchmark model

- Consider a value-maximizing monopolistic all equity-financed firm
- The firm generates uncertain cash flow x_t that is governed by a geometric Brownian motion

$$dx_t = \mu x_t dt + \sigma x_t dz_t$$

- Riskless interest rate is r
  - Investors are assumed to be risk neutral
  - Alternatively, one needs to assume that x can be replicated by a portfolio of traded assets (risk-neutral valuation argument)

Benchmark model

- As x is always positive, the owners of the firm will keep it open forever
- The value of the firm, $V(x)$, can be therefore expressed as

$$V(x) = \frac{x}{r - \mu}$$

which is the standard Gordon (1959) equity valuation formula

Benchmark model

Systematic risk

- Beta of the firm equals the elasticity of the firm value with respect to the level of variable x describing the state of the economy
- In the benchmark case, it equals therefore

$$\beta(x) = \frac{\partial V}{\partial x}\frac{x}{V} = \frac{1}{r-\mu}\underbrace{\frac{x}{\frac{x}{r-\mu}}}_{V(x)} = 1$$

- Beta of the firm equals to the beta of its cash flow x (normalized to 1), as the value of the firm is proportional to x

Benchmark model

Systematic risk

- As we will see, the operating and financing structure of the firm implies that the value of the firm (or, of its equity in particular) is generally not proportional to the variable governing its systematic risk
- Consequently, the firm's equity beta does not usually equal cash (asset) beta
- This fact can have important implications for the expected equity returns

Carlson, Fisher and Giammarino (2004)

- The aim of the paper is to provide a framework for deriving the systematic risk of a firm with
  - growth options, and
  - operating leverage

  explicitly taken into account
- The model is set in the continuous-time framework as the one used throughout the session

Basic model set-up

- Consider a value-maximizing monopolistic all equity-financed firm
- The firm faces economic uncertainty that is modeled as a geometric Brownian motion

$$dx_t = \mu x_t dt + \sigma x_t dz_t$$

Basic model set-up

- Two (sequential) investment options are available
  - No option exercised – *juvenile* firm
  - One option exercised – *adolescent* firm
  - Both options exercised – *mature* firm
- Investment is irreversible
  - This assumption will have implications for operating leverage

Basic model set-up

- Investment cost:
  - I_1 when exercising the first investment option
  - I_2 when exercising the second option
- Fixed operating cost:
  - f_i with i investment options exercised
  - $f_2 > f_1$
- Firm's (gross) revenue:
  - $\Pi_i x$ with i investment options exercised
  - Again, $\Pi_2 > \Pi_1$

Basic model set-up

- Consequently, the operating profit of the firm, $\pi_i(x)$, is:

$$\pi_i(x) = \Pi_i x - f_i$$

- Now, we are in position to calculate the value of each type of firm (*juvenile/adolescent/mature*) as well as to determine its systematic risk and, as a consequence, the expected return

Main results

Mature firm

- Recall that the mature firm has no more growth options available
- The value of the firm, $V_2(x)$, can be therefore expressed as:

$$V_2(x) = \frac{\Pi_2 x}{r - \mu} - \frac{f_2}{r}$$

Main results

Systematic risk

- Beta of firm *i* can be generally expressed as:

$$\beta_i(x) = \frac{\partial V_i}{\partial x} \frac{x}{V_i}$$

which (as mentioned before) equals the elasticity of the firm value with respect to the level of variable *x* describing the state of the economy

  - The paper provides a formal argument for it

Main results

Mature firm

- For a mature firm ($i = 2$), beta equals

$$\beta_2(x) = \frac{\frac{\Pi_2 x}{r-\mu}}{\underbrace{\frac{\Pi_2 x}{r-\mu} - \frac{f_2}{r}}_{V_2(x)}} = 1 + \underbrace{\frac{\frac{f_2}{r}}{V_2(x)}}_{\text{Effect of operating leverage}} > 1$$

- The presence of fixed costs (operating leverage) results in a higher beta of the firm
- Systematic risk decreases as the state of economy improves since $\frac{f_2/r}{V_2(x)}$ decreases with x
- In the limit, $\beta_2(x) = 1$ when $x \to \infty$ and $\beta_2(x) = \infty$ when $V_2(x) \downarrow 0$

Main results

Mature firm

- Empirical implication: firms with higher operating leverage have higher systematic risk and, therefore, higher expected returns
  - This result closely resembles Modigliani and Miller (1958) Proposition II about the required rate of return (cost of capital) of levered equity
- For the proxy for operating leverage and recent empirical evidence, see Novy-Marx (2011) (to follow shortly)

Main results

Juvenile firm

- Consider now a special case of a juvenile firm ($i = 0$) with no assets in place
- Such a firm has two growth options and its value can be written as

$$V_0(x) = x^{\lambda_1} \sum_{j=1}^{2} \frac{\varepsilon_j}{x_j^{\lambda_1}}$$

where $\varepsilon_j = \frac{f_j - f_{j-1} + I_j r}{(\lambda_1 - 1) r}$ and $x_j^{\lambda_1}$ is the standard investment threshold associated with exercising the j-th option

Main results

Juvenile firm

- Beta of the firm equals therefore

$$\beta_0(x) = \underbrace{\frac{\lambda_1 x^{\lambda_1} \sum_{j=1}^{2} \frac{\varepsilon_j}{x_j^{\lambda_1}}}{x^{\lambda_1} \sum_{j=1}^{2} \frac{\varepsilon_j}{x_j^{\lambda_1}}}}_{V_0(x)} = \lambda_1 > 1$$

- The presence of growth options and the absence of assets in place results in a beta of the firm being constant and equal to the elasticity of the value of the growth options with respect to variable x describing the state of the economy (λ_1)

Main results

Adolescent firm

- Finally, consider now an adolescent firm ($i = 1$) with some profit generating ability (assets in place) and one growth option left
- The value of the firm can be written as

$$V_1(x) = \frac{\Pi_1 x}{r - \mu} - \frac{f_1}{r} + x^{\lambda_1} \frac{\varepsilon_2}{x_2^{\lambda_1}}$$

where the first two components reflect the present value of the expected profit flow in the absence of growth options, whereas the third component corresponds to the value of the growth option

Main results

Adolescent firm

- Beta of the firm equals now

$$\beta_1(x) = \underbrace{\frac{\frac{\Pi_1 x}{r-\mu} + \lambda_1 x^{\lambda_1} \frac{\varepsilon_2}{x_2^{\lambda_1}}}{\frac{\Pi_1 x}{r-\mu} - \frac{f_1}{r} + x^{\lambda_1} \frac{\varepsilon_2}{x_2^{\lambda_1}}}}_{V_1(x)} = 1 + \underbrace{(\lambda_1 - 1)\frac{x^{\lambda_1} \frac{\varepsilon_2}{x_2^{\lambda_1}}}{V_1(x)}}_{\text{Effect of growth option}} + \underbrace{\frac{\frac{f_1}{r}}{V_1(x)}}_{\text{Effect of operating leverage}} > 1$$

which is one plus $(\lambda_1 - 1)$ times the ratio of the growth option value and the firm value plus the ratio of the PV of fixed operating costs and the firm value (operating leverage)

Firm value and the state of economy

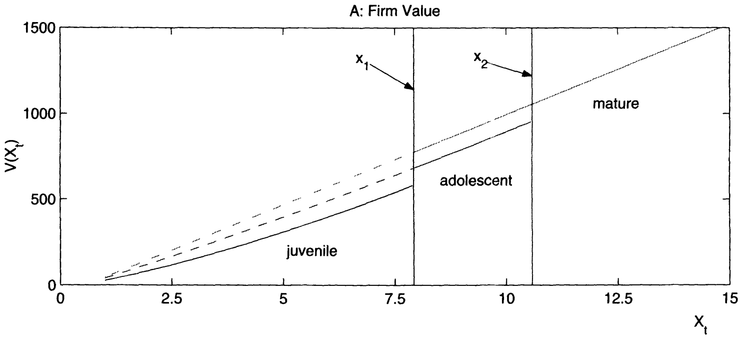


Figure: The relationship between the firm value and the state of economy for parameter values (original paper's notation):
$r = 0.05, \sigma = 0.2, \delta = 0.03, \gamma = 0.5, Q_0 = 1, Q_1 = 5, Q_2 = 10,$
$f_0 = 1, f_1 = 2, f_2 = 3, \lambda_1 = 100, \lambda_2 = 100, K_0 = 100, K_1 = 200,$ and $K_2 = 300$

Firm value and the state of economy

- For low x, three types coexist
  - juvenile firms (here with some assets in place, solid line) [NOTE: This differs from that earlier assumption that the juvenile firm consists of growth options only]
  - adolescent firms (dashed line)
  - mature firms (dashed-dotted line)
- When demand hits threshold $x = x_1$ for juvenile investment, value jumps discretely by the investment amount and valuation changes to that of an adolescent firm
  - Thus, in the region between x_1 and x_2, only two firm types exist
- A similar change occurs when adolescent firms invest at $x = x_2$ and become mature
  - Therefore, for $x > x_2$, only mature firms exist

Systematic risk and firm size

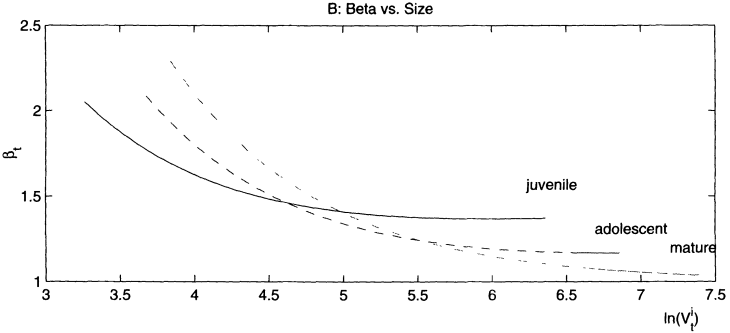


Figure: The relationship between systematic risk and firm size for parameter values (original paper's notation):
$r = 0.05, \sigma = 0.2, \delta = 0.03, \gamma = 0.5, Q_0 = 1, Q_1 = 5, Q_2 = 10, f_0 = 1, f_1 = 2,$
$f_2 = 3, \lambda_1 = 100, \lambda_2 = 100, K_0 = 100, K_1 = 200,$ and $K_2 = 300$

Systematic risk and firm size

- For a given stage in the life of a firm, as firm value increases operating leverage drops
  - This causes risk to decrease
- At the same time, the proportion of growth options in total firm value also increases
  - This effect contributes to an increase in risk
- The combined effect differs across the firm types
  - For *mature* firms, only the operating leverage effect is present and beta decreases in size
  - For *juvenile* and *adolescent* firms, the growth option effect becomes dominant for a small range of x just before investment occurs and the relationship between firm size and beta is reversed (the effect cannot be seen very clearly on the graph though)

Novy-Marx (2011)

- The paper demonstrates that the book-to-market ratio can serve as an empirical proxy for operating leverage
- It is shown that operating leverage can predict cross-sectional returns
- One of the paper's results is that strategies formed by sorting on operating leverage earn significant excess returns

Novy-Marx (2011)

- In particular,
  - intra-industry differences in book-to-market are driven by differences in operating leverage within an industry, resulting in the presence expected return differences
  - inter-industry differences in book-to-market are driven by differences in the capital intensity of production across industries and are unrelated to returns

Operating leverage

	Value-weighted results					Equal-weighted results				
	r^e	α	MKT	SMB	HML	r^e	α	MKT	SMB	HML
Panel A: Intra-industry portfolios										
Intra-industry BM quintiles										
Low	0.52	0.01	1.03	−0.07	−0.18	0.48	−0.21	1.05	0.57	−0.14
	[2.94]	[0.41]	[191]	[−8.49]	[−23.0]	[2.23]	[−3.74]	[96.8]	[32.7]	[−9.12]
2	0.60	0.02	0.97	−0.02	0.04	0.75	−0.04	1.03	0.57	0.14
	[3.43]	[0.61]	[170]	[−1.99]	[4.39]	[3.46]	[−0.87]	[117]	[41.0]	[10.6]
3	0.68	−0.03	1.03	0.01	0.27	0.91	−0.00	1.02	0.71	−0.35
	[3.54]	[−0.64]	[137]	[0.53]	[24.8]	[3.91]	[−0.03]	[129]	[55.6]	[30.7]
4	0.74	0.03	0.98	0.11	0.28	1.10	0.06	1.04	0.89	0.54
	[3.85]	[0.63]	[93.2]	[6.36]	[18.3]	[4.29]	[1.29]	[112]	[59.8]	[40.4]
High	0.87	0.07	1.04	0.23	0.36	1.43	0.24	1.05	1.13	0.75
	[4.17]	[1.46]	[112]	[15.3]	[27.0]	[2.20]	[0.96]	[46.8]	[−1.47]	[3.48]
H-L	0.35	0.06	0.01	0.30	0.54	0.95	0.45	0.01	0.56	0.89
	[3.68]	[0.92]	[0.60]	[15.3]	[30.4]	[6.62]	[6.23]	[0.32]	[25.0]	[43.8]

Figure: Excess returns and alphas to portfolios sorted on book-to-market within industries

Operating leverage

Panel B: Industry portfolios

Industry BM quintiles										
Low	0.52	0.01	1.06	−0.02	−0.28	0.83	−0.01	1.12	0.89	−0.07
	[2.77]	[0.24]	[104]	[−1.02]	[−19.0]	[3.31]	[−0.15]	[81.5]	[40.7]	[−3.53]
2	0.58	0.04	0.99	0.07	−0.15	0.82	−0.08	1.05	0.84	0.20
	[3.20]	[0.69]	[94.7]	[3.89]	[−9.73]	[3.37]	[−1.25]	[81.7]	[40.7]	[10.9]
3	0.64	0.07	0.98	−0.02	−0.01	0.93	0.05	0.98	0.80	0.30
	[3.56]	[1.16]	[82.4]	[−0.88]	[−0.70]	[4.03]	[0.72]	[77.1]	[39.3]	[16.2]
4	0.73	0.08	0.94	−0.01	0.22	0.97	0.04	0.99	0.67	0.47
	[4.00]	[1.29]	[76.2]	[−0.46]	[12.3]	[4.15]	[0.52]	[76.1]	[32.09]	[24.7]
High	0.67	−0.16	0.99	0.01	0.60	0.99	−0.11	0.99	0.81	0.81
	[3.26]	[−2.83]	[88.6]	[0.66]	[37.1]	[3.84]	[−1.74]	[78.9]	[40.3]	[44.6]
H-L	0.15	−0.17	−0.07	0.03	0.88	0.17	−0.10	−0.13	−0.08	0.88
	[1.23]	[−2.21]	[−4.71]	[1.16]	[39.6]	[1.29]	[−1.17]	[−7.63]	[−3.10]	[36.1]

Figure: Excess returns and alphas to portfolios sorted on book-to-market across industries

Growth options and operating leverage

- Betas decrease when operating leverage becomes lower
  - Lower fixed operating costs
  - Better state of the economy, as proxied by x
- Expansion options generally increase the firm's systematic risk
  - Better investment opportunities
  - Better state of the economy

Growth options and operating leverage

- The discussion so far has focused on the effect of growth options and operating leverage
  - We have looked at what happens when cash flow parameter x increases
- The effect of debt and exit options also has a profound effect on the systematic risk of a firm's equity
  - Exit options become relevant when the economic situation of the firm deteriorates (x becoming low)
  - Consequently, we will now focus of the equityholders' default option
  - Although not analyzed here, the mechanism of the effect of a liquidation option (an option to sell the assets of the firm in the secondary market) is identical to that of the option to default

Favara, Schroth and Valta (2012)

- The aim of the paper is to provide a framework for calculating the systematic risk of the equity of a firm with
  - debt, and (in particular)
  - an option to default

  explicitly taken into account
- Again, the model is set in a continuous-time framework of Black-Scholes-Merton, Leland (1994), and Dixit and Pindyck (1994)

The model

- The set-up of the model follows very closely Leland (1994)
- Strategic default is modeled in a similar way as in Fan and Sundaresan (2000) and the possibility of renegotiation breakdown is inspired by Davydenko and Strebulaev (2007)
- Similarly as in Carlson et al. (2004), pre-tax cash flow of the firm follows a geometric Brownian motion

$$dx_t = \mu x_t dt + \sigma x_t dz_t$$

The model

- Corporate tax rate is τ
- The firm has outstanding debt and pays perpetual coupon b
- As in Leland (1994), shareholders have an option to default on coupon payments
- Shareholders default at an optimally chosen trigger x_B
- Upon default, debt renegotiation is attempted

The model

Debt renegotiation

- Debt renegotiation may either succeed or fail
  - If renegotiation succeeds, the value of the firm is split between shareholders and creditors according to their bargaining power parameters η and $1 - \eta$
  - If renegotiation fails, the firm is liquidated
  - If the firm is liquidated, creditors receive a fraction $(1 - \alpha)$ of the firm value at default
  - Parameter $\alpha \in [0, 1]$ reflects liquidation costs
  - Following an unsuccessful renegotiation, shareholders receive nothing
- Renegotiation fails with (exogenous) probability q
  - High q is interpreted as the presence of strict debt enforcement procedures

The model

Equity value

- The value of equity is

$$E(x) = (1-\tau)\left[\frac{x}{r-\mu} - \frac{b}{r} + \left(\frac{b}{r} - [1-(1-q)\eta\alpha]\frac{x_B}{r-\mu}\right)\left(\frac{x}{x_B}\right)^{\lambda_2}\right]$$

where λ_2 is the negative root of the fundamental quadratic:

$$\frac{1}{2}\sigma^2\lambda(\lambda-1) + \mu\lambda - r = 0$$

The model

Default trigger

- Default in the model is declared at the point optimal for equityholders, i.e., by setting

$$\left.\frac{\partial E}{\partial x}\right|_{x=x_B} = 0$$

- This yields

$$x_B = \frac{\lambda_2}{\lambda_2 - 1} \frac{b}{1-(1-q)\eta\alpha} \frac{r-\mu}{r}$$

- x_B decreases with the probability of renegotiation failure q and increases with liquidation cost α as well as with shareholders' bargaining power parameter η

The model

Systematic risk

- Analogously as in Carlson et al. (2004), equity beta equals the elasticity of the equity value with respect to cash flow variable x and can be expressed as:

$$\beta(x) = \frac{\partial E}{\partial x}\frac{x}{E} = \frac{(1-\tau)\left[\frac{x}{r-\mu} + \frac{\lambda_2}{1-\lambda_2}\frac{b}{r}\left(\frac{x}{x_B}\right)^{\lambda_2}\right]}{\underbrace{(1-\tau)\left[\frac{x}{r-\mu} - \frac{b}{r} + \frac{1}{1-\lambda_2}\frac{b}{r}\left(\frac{x}{x_B}\right)^{\lambda_2}\right]}_{E(x)}}$$

$$= 1 - \underbrace{\frac{(1-\tau)\frac{b}{r}\left(\frac{x}{x_B}\right)^{\lambda_2}}{E(x)}}_{\text{Effect of default option}} + \underbrace{\frac{\frac{(1-\tau)b}{r}}{E(x)}}_{\text{Effect of leverage}}$$

The model

Systematic risk

- The presence of debt (leverage) results in a higher equity beta
  - This is a standard result (Modigliani and Miller (1958), Proposition II)
  - A motivating example is included in the textbook by Berk and DeMarzo (next slide)

Systematic risk

Example

Estimates of equity betas and market debt-equity ratios for several airline stocks in the fall of 2005 are shown below:

Ticker	Name	Equity Beta	Debt-Equity Ratio	Debt Beta
LUV	Southwest Airlines Co.	1.13	0.15	0.00
ALK	Alaska Air Group, Inc.	1.80	1.06	0.15
SKYW	SkyWest, Inc.	1.69	1.05	0.15
MESA	Mesa Air Group, Inc.	3.27	3.52	0.30
CAL	Continental Airlines, Inc.	3.76	5.59	0.40

Do the large differences in the equity betas of these firms reflect large differences in the market risk of their operations? What approximate beta would you use to evaluate projects in the airline industry?

Figure: Equity betas and leverage

Systematic risk

Example

The market risk of equity is amplified by the firm's leverage. To assess the market risk of the airlines' operations, we should consider their unlevered betas, which we compute using Eq. 14.9:

Ticker	β_E	$E/(E+D)$	β_D	$D/(E+D)$	β_U
LUV	1.13	87%	0.00	13%	0.98
ALK	1.80	49%	0.15	51%	0.96
SKYW	1.69	49%	0.15	51%	0.90
MESA	3.27	22%	0.30	78%	0.95
CAL	3.76	15%	0.40	85%	0.90

While the airlines' equity betas vary considerably, their unlevered betas are similar. Thus the differences in the market risk of their equity are primarily due to differences in their capital structures. Based on this data, an unlevered beta in the range of 0.90–0.98 would be a reasonable estimate of the market risk of projects in this industry.

Figure: Equity betas and leverage

The model

Systematic risk

- The presence of a default option mitigates the positive effect of debt on equity beta
  - Default option serves as an insurance to equityholders, who will not be held liable with their personal wealth if they decide to default on coupon payment and walk away from the firm
  - Since $\lambda_2 < 0$, the mitigating effect of the default option on β increases with x_B:

$$\frac{\partial \beta(x; x_B)}{\partial x_B} < 0$$

The model

Systematic risk

- Consequently, by calculating first-order derivatives of x_B with respect to key parameters: q (likelihood of renegotiation failure), α (liquidation cost), and η (equityholders' bargaining power), one can also determine the effect of those parameters on equity beta
  - Equity beta increases if the likelihood that debt contracts are enforced increases
$$\frac{\partial \beta(x)}{\partial q} > 0$$
  - Equity beta decreases if the liquidation cost increases
$$\frac{\partial \beta(x)}{\partial \alpha} < 0$$
  - Equity beta decreases if equityholders' bargaining power increases
$$\frac{\partial \beta(x)}{\partial \eta} < 0$$

The model

Systematic risk

- Furthermore, by calculating cross derivatives one can determine the effect of the interaction between the key model parameters on equity beta
  - The negative effect on equity beta of a higher liquidation cost is mitigated by a higher likelihood that debt contracts are enforced

$$\frac{\partial^2 \beta(x)}{\partial \alpha \partial q} > 0$$

  - The negative effect on equity beta of a higher equityholders' bargaining power is mitigated by a higher likelihood that debt contracts are enforced

$$\frac{\partial^2 \beta(x)}{\partial \eta \partial q} > 0$$

Testable hypotheses

- H1: Firms in a legal regime with stricter enforcement of debt contracts have a higher equity beta
- H2: Firms with higher liquidation costs or with higher shareholders bargaining power in case of debt renegotiations have a lower equity beta
- H3: The difference in equity beta between firms facing different liquidation costs or shareholders' bargaining power is smaller in countries with stricter enforcement of debt contracts

Empirical results

Size	0.041***	0.032***	0.034***	0.045***	0.035***
	(0.004)	(0.004)	(0.004)	(0.003)	(0.004)
Book-to-Market	0.017	0.050***	0.047***	0.013	0.048***
	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)
Leverage projection	0.109**	−0.076*	−0.098**	0.123***	−0.069
	(0.046)	(0.043)	(0.042)	(0.047)	(0.043)
Renegotiation failure	0.116***	0.104***	−0.063	−0.290***	−0.191*
	(0.027)	(0.026)	(0.048)	(0.101)	(0.114)
Insiders' share	−0.093***	−0.079***	−0.304***		
	(0.027)	(0.028)	(0.064)		
Intangibles (w/o cash)	−0.645***			−1.181***	
	(0.053)			(0.139)	
Intangibles (with cash)		0.233***			−0.081
		(0.050)			(0.126)
Insiders' share × Renegotiation failure			0.395***		
			(0.094)		
Intangibles (w/o cash) × Renegotiation failure				0.927***	
				(0.208)	
Intangibles (with cash) × Renegotiation failure					0.547***
					(0.194)
Constant	0.769***	0.437***	0.670***	0.944***	0.556***
	(0.039)	(0.044)	(0.046)	(0.068)	(0.078)
F statistic	3,126.192	1,612.794	1,653.148	3,115.472	1,593.666
Observations	388,816	388,816	388,816	388,816	388,816
Average adjusted R^2	0.05	0.03	0.02	0.05	0.03

Figure: Equity betas and renegotiation frictions

Empirical results

- As predicted, higher probability of renegotiation failure results in a higher equity beta
- Higher insiders' share and a higher proportion of intangibles result in a lower beta
- Better debt contract enforcement mitigates the effect of a higher insiders' share and of a higher proportion of intangibles

Summary

- Operating leverage (fixed costs) increases equity beta in a similar way the financial leverage does
- Growth options increase equity beta
  - The effect is stronger for more in-the-money options
- Leverage increases equity beta (as predicted in Modigliani and Miller (1958) Proposition II)
- Default option reduces equity beta
  - The effect is stronger if equityholders' payoff upon default is higher

Related literature

- Seasoned equity offerings (SEOs) (Carlson, Fisher, and Giammarino, 2006)
- Leverage (Bhamra, Kuehn, and Strebulaev, 2008; Gomes and Schmid, 2010)
- Mergers and acquisitions (Hackbarth and Morellec, 2008)
- Symmetric (Aguerrevere, 2009) and asymmetric oligopoly (Carlson, Dockner, Fisher, and Giammarino, 2014)
- New ventures (Berk, Green, and Naik, 2004; Garlappi, 2004)

Part IV

Game-theoretical aspects of real options exercise

Introduction

Outline

- Strategic growth option exercise in a duopoly (Smets, 1991; Grenadier, 1996; Huisman and Kort, 1999)
  - Extension for asymmetric firms (Joaquin and Butler, 2000; Pawlina and Kort, 2006)
- Industry exit and the role of debt (Lambrecht, 2001)

Key papers

- Grenadier, S. R. (1996): “The Strategic Exercise of Options: Development Cascades and Overbuilding in Real Estate Markets,” Journal of Finance, 51, 1653–1679
- Pawlina, G., and P. M. Kort (2006): “Real Options in an Asymmetric Duopoly: Who Benefits from Your Competitive Disadvantage?,” Journal of Economics and Management Strategy, 15, 1–35
- Lambrecht, B. (2001): “The Impact of Debt Financing on Entry and Exit in a Duopoly,” Review of Financial Studies, 14, 765–804

Strategic growth option exercise in a duopoly

- In this section, we will study the effects of duopolistic competition on the optimal real option exercise strategies
- Such a modeling approach recognizes non-exclusivity of growth options held by firms in a competitive environment
  - In particular, the optimal exercise decision of a firm competing in an oligopolistic market depends not only on the value of the underlying economic variable but also on the actions undertaken by its competitor(s) (see, for instance, Myers, 1987; Smets, 1991; and Huisman, 2001)

Strategic growth option exercise in a duopoly

- There are 2 firms competing in the same market and which are assumed to face the same source of uncertainty
  - Consequences of relaxing this assumption are studied by Thijssen (2010)
- Uncertainty in each of the firms' profits is introduced via a GBM

$$dx_t = \mu x_t dt + \sigma x_t dz_t$$

- Each firm can make a single profit-enhancing investment, which costs I to undertake
  - We also assume that initial x is sufficiently low so that immediate investment is not optimal

Strategic growth option exercise in a duopoly

- The instantaneous profit of Firm i, $i \in \{1, 2\}$, can be expressed as

$$\pi_{N_i N_j}(x) = x D_{N_i N_j} \tag{10}$$

where, for $k \in \{i, j\}$:

$$N_k = \begin{cases} 0 & \text{if firm } k \text{ has not invested} \\ 1 & \text{if firm } k \text{ has invested} \end{cases}$$

- $D_{N_i N_j}$ stands for the deterministic contribution to the profit function, and it holds that

$$\begin{array}{ccc} D_{10} & > & D_{00} \\ \vee & & \vee \\ D_{11} & > & D_{01} \end{array} \tag{11}$$

Strategic growth option exercise in a duopoly

- $D_{10} > D_{00}$ means that the profit of the firm that invests as first exceeds the initial (symmetric) profit
- Furthermore, this investment leads to a deterioration of the profit of the firm that did not undertake the project yet, i.e. $D_{00} > D_{01}$
- Finally, the investment made by the lagging firm enhances its own profit, so $D_{11} > D_{01}$, but also reduces the profit of the first mover, i.e., $D_{11} < D_{10}$
- Note that by setting $D_{00} = D_{01} = 0$ we would arrive at the new market model analyzed in Huisman (2001), Ch. 8, and Joaquin and Butler (2000)

Strategic growth option exercise in a duopoly

- In general, there are three possibilities concerning the relative timing of the firms' investment:
  1. Firm 1 invests first, i.e., becomes the leader
  2. Firm 2 invest first, which results in Firm 1 becoming the follower
  3. Firms invest simultaneously
- A standard approach used in dynamic games is to solve the problem backwards in time (cf. Fudenberg and Tirole, 1985)
  - We first determine the optimal response of the follower to the strategy taken by the leader
  - We subsequently derive the optimal strategy of the leader
- Finally, we compare value functions under the sequential investment scenario with the values that firms can obtain when investing simultaneously

Value function of the follower

- Consider the investment decision of the follower in a situation where the leader has already invested
- As the leader takes no further action, the problem of the follower is identical to the well-known non-strategic investment problem
- The optimal investment threshold calculated using the standard argument is given by

$$x^F = \frac{\lambda_1}{\lambda_1 - 1} \frac{I(r-\mu)}{D_{11} - D_{01}}$$

Strategic growth option exercise in a duopoly

- The value of a firm as the follower equals

$$V^F(x) = \frac{xD_{01}}{r-\mu} + \left(\frac{x^F (D_{11} - D_{01})}{r-\mu} - I \right) \left(\frac{x}{x^F} \right)^{\lambda_1}$$

- The value function of the follower consists of two components
  - the present value of the current level of profits
  - the NPV of the follower's profit-enhancing investment discounted using the probability weighted discount factor stochastic discount factor associated with reaching threshold x^F when starting from current level x

Value function of the leader

- Now, we are ready to introduce the value function of a firm as the leader
  - The function is evaluated assuming immediate investment as the choice between between becoming the leader immediately and waiting to be the follower will be the relevant one

$$V^L(x) = \frac{xD_{10}}{r-\mu} - I - \frac{x^F(D_{10} - D_{11})}{r-\mu}\left(\frac{x}{x^F}\right)^{\lambda_1}$$

- The value function of the follower consists of two components
  - the NPV of immediate investment
  - the (discounted) PV of the profit reduction associated with the follower's future investment

Value function under simultaneous investment

- It is possible that the firms invest simultaneously in equilibrium
- The value function of a firm when investing simultaneously with its competitor at threshold x^S is

$$V^S(x) = \frac{xD_{00}}{r-\mu} + \left(\frac{x^S(D_{11}-D_{00})}{r-\mu} - I\right)\left(\frac{x}{x^S}\right)^{\lambda_1}$$

- The optimal threshold of simultaneous investment (calculated using the standard argument) is

$$x^S = \frac{\lambda_1}{\lambda_1 - 1}\frac{I(r-\mu)}{D_{11}-D_{00}}$$

Sequential investment equilibrium

- Let's first consider the equilibrium in which firms invest sequentially
- Each firm has to take into account the fact that its competitor aims to preempt it as soon as a certain threshold is reached
- This threshold, denoted by x^P, is the lowest realization of the process x for which each firm is indifferent between investing immediately and becoming the leader and waiting to become the follower

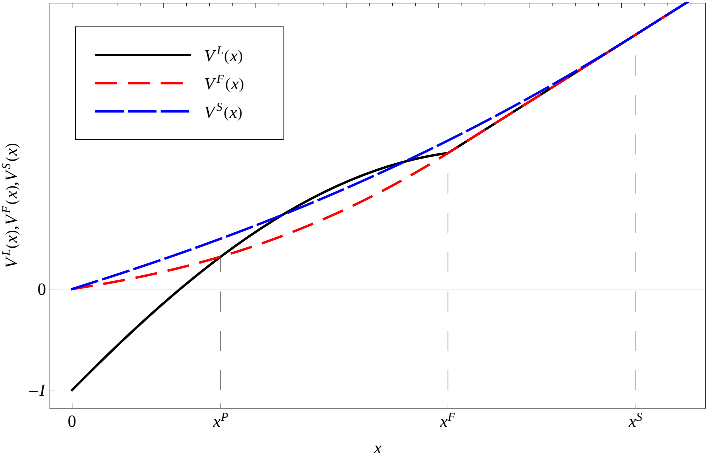
Sequential investment equilibrium

- x^P is the (smallest) solution to

$$V^L(x) - V^F(x) = 0$$

- As a result, the leader invests at x^P and the follower waits until x^F is reached
- Each firm's attempt to preempt its competitor leads to rent equalization
- In equilibrium Firm 1 (Firm 2) becomes the leader (the follower) with probability 0.5

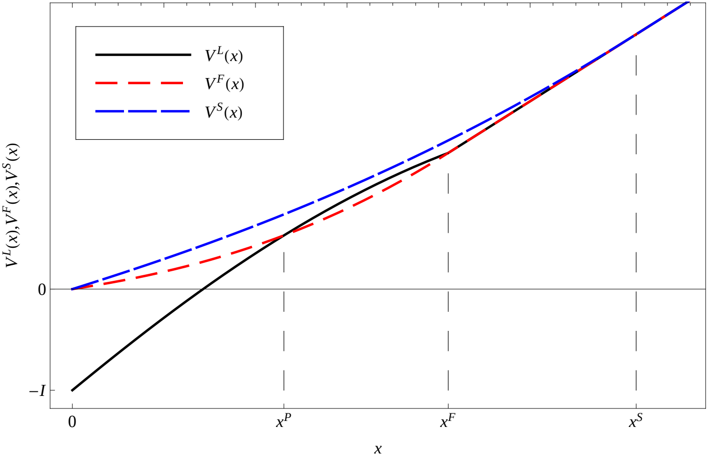
Sequential investment equilibrium (with preemption threat)



Simultaneous investment equilibrium

- In the simultaneous investment equilibrium, firms invest at the same point in time
- This type of equilibrium is only possible if firms already compete in the market and they face the risk of cannibalizing their profits from existing assets
- Otherwise, the “tacit collusion” would not be sustainable since the threat of losing existing profits would not exist (as in Huisman, 2001, Ch. 8, and Joaquin and Butler, 2000)

Simultaneous investment equilibrium

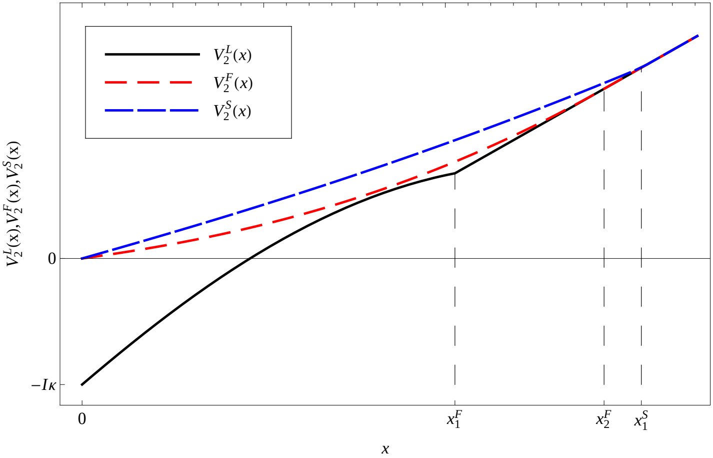


Asymmetric firms

- Introducing some aspect of asymmetry between firms may lead to an additional equilibrium (cf. Huisman, 2001; Pawlina and Kort, 2006)
- If one of the firms faces sufficiently high disadvantage (e.g., due to a higher required investment outlay $I' > I$), its competitor becomes the leader without facing the preemption threat
- The optimal leader's investment threshold of the firm with the cost advantage is then simply

$$x^L = \frac{\lambda_1}{\lambda_1 - 1} \frac{I\,(r - \mu)}{D_{10} - D_{00}}$$

Sequential investment equilibrium with no preemption threat



Conditions for equilibria

- There exists a unique value of $I'/I \equiv \kappa > 1$, denoted by κ^*, which is equal to

$$\kappa^* = \left(\frac{w^{\lambda_1} - 1}{\lambda_1 (w - 1)} \right)^{\frac{1}{\lambda_1 - 1}}$$

where

$$w = \frac{D_{10} - D_{01}}{D_{11} - D_{01}}$$

which separates the regions of the preemptive and the sequential equilibrium, conditionally on one of them occurring

- For $\kappa < \kappa^*$ Firm 1 needs to take into account possible preemption by Firm 2

- $\kappa \geq \kappa^*$ implies that firms always invest sequentially at their optimal thresholds

Conditions for equilibria

- There exists a unique value of $\kappa \geq 1$, denoted by κ^{**}, which is equal to

$$\kappa^{**} = \max\left(v \left(\frac{\lambda_1 (u-1)}{u^{\lambda_1} - 1} \right)^{\frac{1}{\lambda_1 - 1}}, 1 \right)$$

  where

$$u = \frac{D_{10} - D_{00}}{D_{11} - D_{00}} \qquad \text{and} \qquad v = \frac{D_{11} - D_{01}}{D_{11} - D_{00}}$$

  which determines the regions of the simultaneous and the sequential/preemptive investment equilibria

- For $\kappa < \kappa^{**}$ the resulting equilibrium is of the joint investment type

- For $\kappa \geq \kappa^{**}$ the sequential/preemptive investment equilibrium occurs

Conditions for equilibria

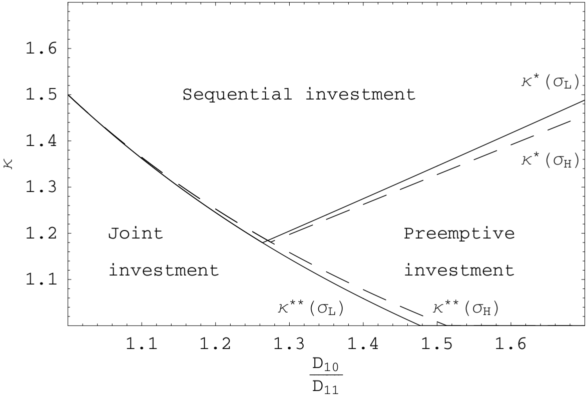


Figure: Equilibrium regions for parameter values: $r = 0.05$, $\mu = 0.015$, $\sigma_L = 0.05$, $\sigma_H = 0.25$, $D_{00} = 0.5$, $D_{01} = 0.25$, and $D_{11} = 1$

Digression #5: The effect of subsequent contingencies on the optimal exercise policy

- If an investment project has two or more stages, the timing of each stage can be determined individually
  - In other words, the existence of stage 2 has no effect on the optimal exercise time of stage (cf. Dixit and Pindyck, 1994)
- In a similar way, an investor who must take into account subsequent investments of the competitors employs the same investment policy as a monopolist who is not threatened by such future events
  - This result is due to the special structure of optimal stopping problems and is demonstrated in Leahy (1993) and Baldursson and Karatzas (1997)

The effect of subsequent contingencies on the optimal exercise policy

- A similar principle can be applied to a sequence of real options associated with a downward movement of the GBM
- Note, that the additional options cannot be ignored if one is associated with an upward and another with a downward movement of the GBM (or vice versa)
  - For example, the presence of a subsequent liquidation option is relevant for the timing of the exercise of an investment option

Exit in a duopoly

- An opposite decision to investment is industry exit
- Imperfect competition results in the exit behavior of a firm being influenced by (anticipated) actions of its competitor(s)
- Firm may engage in "war of attrition" hoping that it is a competitor that will exit first
  - The problem of exit in a duopoly is analyzed in pure strategies, e.g., in
    - Lambrecht (2001)
    - Murto (2004)

    whereas mixed strategies are considered in, among others,
    - Khadem and Perraudin (2000)
    - Miltersen and Schwartz (2007)

Exit in a duopoly

Lambrecht (2001)

- Lambrecht (2001) analyzes the effect of debt financing on exit (and entry; we will entirely focus on the former) in a duopoly
- One of results of the paper is that the order of exit of firms may not only depend on their own characteristics but also on the common economic factors
  - This outcome is a consequence of the fact that firms differ along two dimensions: leverage and profitability
  - If each firm dominates in only one of the two dimensions, the overall dominance (i.e., the relative importance of each of the criteria) is a function of economic conditions, captured by μ, σ and r

Exit in a duopoly

- Consider first the setup assuming that a firm is a monopolist
- Instantaneous profit of a monopolist is Πx_t, where is Π a constant and x_t follows the following GBM:

$$dx_t = \mu x_t dt + \sigma x_t dz_t$$

- The firm is assumed to have a perpetual debt with an instantaneous coupon b

Exit in a duopoly

- The (non-strategic) value of the firm's equity E and debt D equals

$$\begin{aligned} E(x) &= \frac{\Pi x}{r-\mu} - \frac{b}{r} + \left(\frac{b}{r} - \frac{\Pi \underline{x}}{r-\mu}\right)\left(\frac{x}{\underline{x}}\right)^{\lambda_2} \\ D(x) &= \frac{b}{r} - \left(\frac{b}{r} - (1-\alpha)V(\underline{x})\right)\left(\frac{x}{\underline{x}}\right)^{\lambda_2} \end{aligned}$$

where

$$\underline{x} = \frac{-\lambda_2}{1-\lambda_2}\frac{b(r-\mu)}{\Pi r}$$

is the optimal bankruptcy threshold determined by equityholders

- When competitive interactions are ignored (as above), the model is identical to Leland (1994)

Exit in a duopoly

- Now, we allow for the presence of two firms in the market, each receiving profit flow (net of coupon) of $\pi_k x - b_k$, $k \in \{i, j\}$
- Assume that it is Firm j that is expected to declare bankruptcy as first (it becomes a *l*oser; then Firm i becomes a *w*inner)
- Then as long as both firms still operate in the industry ($x > \underline{x}_{jd}$), we have

$$E_{iw}(x) = \frac{\pi_i x}{r-\mu} - \frac{b_i}{r} + \left(\frac{x}{\underline{x}_{jd}}\right)^{\lambda_2} \left[\frac{(\Pi_i - \pi_i)\,\underline{x}_{jd}}{r-\mu} + \left(\frac{b_i}{r} - \frac{\Pi_i \underline{x}_{im}}{r-\mu}\right)\left(\frac{\underline{x}_{jd}}{\underline{x}_{im}}\right)^{\lambda_2}\right]$$

with

$$\begin{aligned} \underline{x}_{jd} &= \frac{-\lambda_2}{1-\lambda_2}\frac{b_j(r-\mu)}{\pi_j r} \\ \underline{x}_{im} &= \frac{-\lambda_2}{1-\lambda_2}\frac{b_i(r-\mu)}{\Pi_i r} \end{aligned}$$

Exit in a duopoly

- Once Firm j exits ($x \leq \underline{x}_{jd}$), the value of Firm i's equity is given by

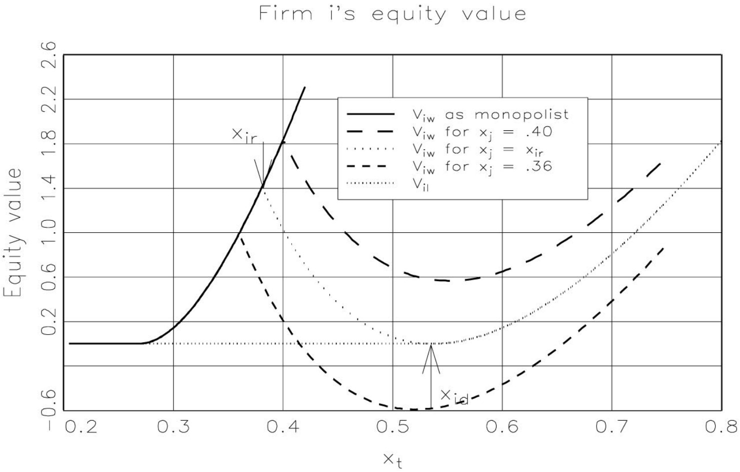
$$E_{iw}(x) = \frac{\Pi_i x}{r-\mu} - \frac{b_i}{r} + \left(\frac{b_i}{r} - \frac{\Pi_i \underline{x}_{im}}{r-\mu}\right)\left(\frac{x}{\underline{x}_{im}}\right)^{\lambda_2}$$

- For any $x > \underline{x}_{im}$, the value of Firm i's debt equals

$$D_{iw}(x) = \frac{b_i}{r} - \left(\frac{b_i}{r} - (1-\alpha) V_i(\underline{x}_{im})\right)\left(\frac{x}{\underline{x}_{im}}\right)^{\lambda_2}$$

Exit in a duopoly

Lambrecht (2001)



Exit in a duopoly

- Denote by $\underline{x}_{ir}$ the threshold level of the stochastic process such that Firm i is indifferent between exiting first and exiting second given that Firm j exits at $\underline{x}_{ir}$

$$\underline{x}_{ir} = \left[\frac{b_i \left(r - \mu \right)}{\left(\Pi_i - \pi_i \right) r \left(1 - \lambda_2 \right)} \left(\underline{x}_{id}^{-\lambda_2} - \underline{x}_{im}^{-\lambda_2} \right) \right]^{\frac{1}{1-\lambda_2}}$$

- If Firm i strictly dominates Firm j ($\underline{x}_{ir} \leq \underline{x}_{jm}$ or $\underline{x}_{id} \leq \underline{x}_{jr}$), then there is a unique Nash equilibrium whereby Firm j leaves first
- If neither firm strictly dominates the other ($\underline{x}_{jm} \leq \underline{x}_{ir} \leq \underline{x}_{jd}$ and $\underline{x}_{im} \leq \underline{x}_{jr} \leq \underline{x}_{id}$) then there are two Nash equilibria
  - Only one of the two Nash equilibria is subgame perfect, namely the equilibrium which involves the firm with the highest monopoly exit threshold (say, Firm j) leaving first at $\underline{x}_{jd}$
  - Then Firm i leaves at $\underline{x}_{im}$

Exit in a duopoly

- Lambrecht (2001) shows that Firm j leaves the market as first if and only if $\underline{x}_{id} \leq \underline{x}_{jr}$ or $\underline{x}_{im} \leq \underline{x}_{jm}$ and $\underline{x}_{ir} \leq \underline{x}_{jd}$, or, equivalently, if

$$\frac{b_i}{b_j} < \max\left\{\pi_i\left(\frac{\pi_j^{\lambda_2} - \Pi_j^{\lambda_2}}{-\lambda_2\left(\Pi_j - \pi_j\right)}\right)^{\frac{1}{1-\lambda_2}}, \min\left\{\frac{\Pi_i}{\Pi_j}, \frac{1}{\pi_j}\left(\frac{-\lambda_2\left(\Pi_i - \pi_i\right)}{\pi_i^{\lambda_2} - \Pi_i^{\lambda_2}}\right)^{\frac{1}{1-\lambda_2}}\right\}\right\}$$

Summary

- The presence of competitive interactions reduces growth option value
  - Rent equalization between the leader and the follower occurs in the case of symmetric firms
  - Firm may non-cooperatively choose to invest late to maximize the value of their expansion options
  - A sufficiently high degree of asymmetry between firms makes the leader act as a monopolist
- Debt interacts with the firm profitability to determine the order of industry exit in a duopoly